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APPLIED MECHANICS REVIEWS

VOL. 6, NO. 10 MARTIN GOLAND *Editor* OCTOBER 1953

THERMODYNAMIC EQUATIONS OF STATE

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THERMODYNAMICS is concerned with states and behavior of continuous media as related primarily to thermal and mechanical actions. The media are involved in systems which may be described quantitatively by state variables. Organized empirical evidence in the form of thermodynamic laws, as well as developments from physical theories regarding the microscopic discontinuous structure of matter, indicate the existence of equations of state relating these variables.

Thermodynamic states are of two types, thermostatic and thermokinetic. Thermostatic states are those existing under conditions of thermodynamic equilibrium, and are described by the thermostatic properties of the system. These comprise conventional thermodynamic properties.

Under nonequilibrium conditions, a system may be said to exist in a thermokinetic state. Nevertheless, thermostatic properties for such a system may be established either locally or extensively by a process of thermostatic isolation (1).[1] In addition, properties characteristic of the thermokinetic state itself may be defined and are useful in such fields as transport phenomena and chemical kinetics.

A fluid may be defined as a continuous medium which, when acted on by any shearing stress, cannot remain in static equilibrium. Accordingly, pressure describes the mechanical or hydrostatic state, and pressure together with temperature suffice to describe the thermostatic state. The state of nonfluid media may also be identified by pressure and temperature under certain restricted conditions of stress.

The physical equation of state correlates the primary thermostatic state variables: density, pressure, and temperature. The thermal equations of state relate the energy functions to the primary variables. These functions are dependent on each other and on the physical equation of state as indicated by the typical set of thermodynamic state relations

$$V = \frac{\partial F}{\partial P}\Big)_T, \quad S = -\frac{\partial F}{\partial T}\Big)_P$$

$$H = F + TS = F - T\frac{\partial F}{\partial T}\Big)_P$$

$$U = H - PV = F - T\frac{\partial F}{\partial T}\Big)_P - P\frac{\partial F}{\partial P}\Big)_T$$

$$C_P = -T\frac{\partial^2 F}{\partial T^2}\Big)_P$$

[1] Numbers in parentheses indicate References at end of paper.

A systematic discussion of the computation of the thermostatic properties of real gases and mixtures of real gases from the physical equation of state and other basic sources of information is given by Beattie (2).

The properties of a mixture are derivable from those of the component pure substances only if the mixture is an ideal solution. Since this circumstance occurs rarely, composition must be included generally as a fundamental state parameter.

The over-all quantitative thermokinetic state of a nonequilibrium system is described by the rate of entropy production. The differential rate of entropy change per unit mass is given by

$$\frac{ds}{dt} = -\frac{1}{\rho}\nabla\cdot s + \sigma$$

where s is the entropy transport vector derived from reversible heat and mass transfer, and σ is the irreversible entropy production (1). The latter is the sum of (a) an extrinsic entropy production σ_e, arising from the effect of interactions such as result from temperature, velocity, and concentration gradients; and (b) an intrinsic entropy production σ_i, arising from a thermodynamic unbalance between the intrinsic components of a system such as the reactants and products in a chemical reaction or the rotational-vibrational and translational degrees of freedom in the thermal relaxation of a rapidly expanding gas.

The extrinsic entropy production can always be written essentially as the sum of products of effects such as heat flow, viscous stress, mass diffusion, and the corresponding gradients of temperature, velocity, and chemical potential (1, 3, 4)

$$\sigma_e = \frac{1}{T}\left(-\bar{q}\cdot\nabla\ln T + \bar{\tau}\cdot\nabla V - \sum_k \bar{m}_k\cdot\nabla\mu_k\right)$$

The effects themselves depend on the gradients as conventionally described by principles such as Fourier's law of heat conduction, Newton's law of viscosity, and Fick's law of diffusion.

For most systems and substances, the effects are linearly dependent on the gradients. The coefficients in these linear relations, suitably reduced, characterize the thermokinetic action. Accordingly, they may be identified as thermokinetic properties. These, in particular, from their association with the gradient-induced effects are also known as transport properties. The most familiar among them are the thermal conductivity, viscosity, and diffusion coefficients.

The intrinsic entropy production is similarly the sum of products of reaction rates and corresponding driving forces. Its

formulation is generally in terms of coefficients such as rate constants and relaxation times (5). These, as reaction properties, together with the transport properties, comprise the thermokinetic state properties. These properties are usually functions only of the primary thermostatic properties.

An expression for the entropy production σ, in terms of the several gradients and reaction rates with thermokinetic properties occuring as thermostatic state dependent coefficients, represents the general thermokinetic equation of state.

The physical equation of state for gases has been extended beyond the perfect gas relation by many empirical correlations. The most successful and widely used of these are the van der Waals

$$(P + a\rho^2)(1 - b\rho) = \rho RT$$

the Beattie-Bridgeman (6), giving a compressibility factor

$$Z = 1 + \left(B_0 - \frac{A_0}{RT} - \frac{C}{T^3}\right)\rho + \left(-B_0 b + \frac{A_0 a}{RT} - \frac{B_0 C}{T^3}\right)\rho^2 + \frac{B_0 bc}{T^3}\rho^3$$

and the Benedict-Webb-Rubin (7) with compressibility factor

$$Z = 1 + \left(B_0 - \frac{A_0}{RT} - \frac{C_0}{RT^3}\right)\rho + \left(b - \frac{a}{RT}\right)\rho^2 + \frac{a\alpha}{RT}\rho^5 + \frac{C}{RT^3}(1 + \gamma\rho^2)\rho^2 e^{-\gamma\rho^2}$$

in order of increasing scope and accuracy. Each of these fits a certain appropriate range with good accuracy depending on a suitable choice of constants. The van der Waals constants are normally though not necessarily determined by conditions at the critical point. Constants for the Beattie-Bridgeman and Benedict-Webb-Rubin equations are given respectively by (6) and (7, 8, 9).

Current interest with respect to gases is in a higher level of empiricism derived from the dependence of the equation of state on the intermolecular force potential. Semi-empirical formulations for this potential for bimolecular interactions have successfully predicted low-order virial coefficients. The Lennard-Jones potential for nonpolar gases (10) and the Stockmayer potential for polar gases (11) have been widely adopted. The virial coefficient development based on these has been given by Hirschfelder and associates (12, 13). These results have been applied to the systematic tabulation of imperfect gas properties of air, oxygen, and nitrogen by Hall and Ibele (14, 15) and the National Bureau of Standards (16).

The law of corresponding states has provided graphical means of estimating gas properties beyond the range of empirical or semi-theoretical formulation. Charts have been prepared by Dodge, Su, and Obert (17, 18, 19). In its fundamental form this law presumes that a universal relation

$$F\left(\frac{P}{P_c}, \frac{T}{T_c}, \frac{RT_c}{P_c}\rho\right) = 0$$

exists. For moderately sized nonpolar gases a consistency within several per cent is observed. In general, corrections are essential. For light gases such as helium and hydrogen, the addition of a quantum parameter $\Lambda^* = h/(m\epsilon_0 r_0^2)^{1/2}$ provides satisfactory correlation (20). For polar gases such as the freons, water, and ammonia a dipole moment parameter $\mu^* = \mu^2/\epsilon_0 r_0^3$ must be added (21). For heavier and nonspherical molecules such as many complex organic gases, size and shape factors enter (22).

For dense gases and liquids, satisfactory semi-theoretical expressions for the physical equation of state are unavailable. The free volume concept implicit in the van der Waals $(v - b)$ term has been extended to this region with some success (23, 24). Exploratory studies have appeared extending the intermolecular force analysis to high densities (25).

The thermal equation of state is basically dependent on certain key items such as specific heats, heats of vaporization and sublimation, and heats of formation. Beattie (2) provides a survey of relations involved. A collection of key data and constants for many substances has been prepared by the Bureau of Standards (26).

A comprehensive treatise on the thermostatic equations of state for fluids is given by Beattie, Stockmayer, and Kimball in (27).

The thermokinetic equation of state as described by the transport and reaction properties remains largely dependent on coordinated experimental data. Viscosity and thermal conductivity for pure substances are the best known of the transport properties. These two have been investigated theoretically for gases, extending the Sutherland type of equation by reference to intermolecular potentials (13, 28, 29). Diffusion coefficients for various mixtures are gradually being determined. Little data have appeared on thermodiffusion coefficients. Reviews on transport properties have been prepared by Hawkins (30), Johnson (31), and Hilsenrath and Touloukian (32).

Reaction properties of mixtures form a key element in the domain of chemical kinetics. Reference to the literature of the physical chemist and the chemical engineer will reveal an extensively developed art and science (33, 34). Data on chemical kinetics have been collected by the Bureau of Standards (35).

For pure substances, reaction properties occur only exceptionally. In the rapid expansion of a gas, the relaxation time measures the rate of adjustment to equilibrium for the translational-rotational and vibrational degrees of freedom (36). Similar considerations seem to apply in analyzing the thermal conductivity and the second viscosity coefficient of fluids.

The thermodynamics of solids is still essentially the thermostatics of pure crystals. The applicability of thermokinetics, except for the trivial cases of thermal conductivity and solid diffusion, is yet to be critically considered.

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Theoretical and Experimental Methods

2971. **Kirpichev, V. L., Essays on mechanics [Besedí o mekhanitse],** Moscow-Leningrad, Gosud. Izdat. Tekh.-Teor. Lit., 1951, 360 pp..

This fifth edition of the book is designed for everyone interested in theoretical mechanics and will improve his knowledge of this science. Many important phases of mechanics are often studied somewhat superficially and cannot then be applied with success to problems of technical practice. In a pleasant way, author treats in 17 essays just those facts which usually are not well understood by students.

As for the separate parts of the work, we expressly point to excellent sections on the principle of virtual displacements controlling the equilibrium of mechanical systems and forming the basis for the development of analytical mechanics. Then come considerations about d'Alembert's principle, this magnificent extension of the principle of virtual work from statics to dynamics. As an illustration of d'Alembert's principle, the general laws of dynamics are used—the motion of the center of mass, principle of angular momentum, of conservation of energy, and others. Attention is given to problems on rotation of a rigid body about a fixed point (motion of gyroscopes). The two final chapters deal with the case of impact and, more generally, with instantaneous forces in mechanics. The book ends with some statements about theory of dynamical models.

We warmly recommend the work to all engineers.

V. Vodička, Czechoslovakia

2972. **Golubev, V. V., Lectures on analytical theory of differential equations [Lektsiĭ po analiticheskoi teorii differentsial'nikh uravneniye],** Moscow-Leningrad, Gosud. Izdat. Tekh.-Teor. Lit., 1950, 436 pp..

This book, intended for students of mechanics in Russian universities, treats difficult problems which, for the most part, have been scattered in various periodicals.

The analytical theory of differential equations studies the integrals of these equations without directly calculating them, making use of the methods of functions of a complex variable. This leads in certain cases to special classes of functions, such as elliptic, automorphic, etc.

Also of particular interest are the results relating to the methods of integrating differential equations by series based upon considerations of fixed and mobile singularities of such equations, rigorous theoretical grounds of conformal representation, and many other topics which can only be located with difficulty in the literature.

Golubev's excellent work deals with all of these subjects. The attention of engineers and applied physicists is especially called to chapters 3–5 which treat the theory of second-order equations, and to chapter 6 which gives exact foundations for conformal representation.

The reviewer considers this book to be the most successful of its kind and recommends it to engineers, physicists, and mathematicians.

V. Vodička, Czechoslovakia

2973. Pucci, C., Bounds for solutions of Laplace's equation satisfying mixed conditions, *J. rational Mech. Analysis* **2,** 2, 299–302, 1953.

General boundary condition $\alpha(du/dl) + \beta u = \gamma$ for Poisson equation $\nabla^2 u = f$ is considered. Appropriate boundary and interior inequalities are deduced for a pair of functions w_1 and w_2 in order to insure that $w_1 \leqslant u \leqslant w_2$. A simple example is provided for which $u = 1$ on part of the boundary and $\partial u/\partial n = 0$ on the remainder.

Reviewer was confused by statement to the effect that if a function attains its minimum on boundary then its inward derivative will be nonpositive. Opposite result seems to be incorporated in proof.

S. H. Crandall, USA

2974. Hartman, P., On the derivatives of solutions of linear, second order, ordinary differential equations, *Amer. J. Math.* **75,** 1, 173–177, Jan. 1953.

In the differential equation $d^2y/dt^2 + qy = 0$, an estimate is obtained for dy/dt in terms of y, where $q(t)$ is continuous for $0 \leqslant t < \infty$.

A. R. Mitchell, Scotland

2975. Seifert, G., On certain solutions of a pendulum-type equation, *Quart. appl. Math.* **11,** 1, 127–131, Apr. 1953.

Differential equations of type $d^2y/dt^2 + f(y)\cdot dy/dt = g(y)$, where $f(y)$ and $g(y)$ are functions of period 2π in y, are involved in studies of stability properties for various physical systems. Among names associated with this equation are Liénard, Levinson, authors cited in paper under review, and a number of Russian mathematicians whose papers are reviewed in recent issues of *Mathematical Reviews*. The present paper studies this equation in connection with the oscillations of a synchronous motor around its average angular velocity. The essential variation from known cases consists of permitting the function $g(y)$ to have a finite number of roots in each period instead of requiring it to be nonzero and of fixed sign. A set of conditions on the coefficients is developed under which the equation has a periodic solution of the second kind, i.e., a solution $y(t)$ whose derivative is a periodic function of y with period 2π for all values of t. These conditions are given in forms readily adaptable to the calculus techniques used in the proofs. The author indicates that certain modifications in these might increase the ease of their application.

W. M. Whyburn, USA

2976. Egloff, W., On an eigenvalue problem of potential theory related to the theory of spherical deformations (in German), *Math. Nachr.* **8**, 99–122, 1952.

Domains D on the surface of a sphere are to be determined, for which there exists an infinitesimal isometric deformation moving all points on the boundary of D parallel to the equatorial plane of the sphere. Liebmann in 1920 showed the existence of a discrete set of circular domains D on the sphere with this property. In general, one can reduce the problem by stereographic projection from the south pole to a "third boundary-value problem" for the potential equation in the image domain D' of D. (A linear combination of the function and its first derivatives has to vanish on the boundary.) The solution of this problem leads to a homogeneous Fredholm equation. The author discusses the existence of solutions of this problem for the special case where D' is an ellipse. Using a Fourier series expansion, the integral equation goes over into a rather complicated system of difference equations for the coefficients. The determination of eigenvalues is achieved by means of continued fractions. A numerical scheme is discussed for finding an ellipse D' of given shape for which the problem has a solution.

Courtesy of Mathematical Reviews F. John, USA

2977. Duff, G. F. D., Boundary value problems associated with the tensor Laplace equation, *Canad. J. Math.* **5**, 2, 196–210, 1952.

Uniqueness theorems for boundary-value problems are developed in N-dimensional Riemannian spaces. Close analogy is maintained with three types of problem familiar in classical potential theory: Dirichlet's, Neumann's, and mixed-type boundary conditions. No immediate applications to engineering problems are indicated, but the work appears basic to development of generalized potential theory. W. Bend, USA

2978. Stewart, C. A., Fourier expansions, *Proc. Glasgow math. Assoc.* **1**, 76–93, 1952.

Beginning with linear differential equations with constant coefficients, author employs step functions, Laplace transforms, contour integrals, and residue theory to establish the representation of a function of bounded variation on the interval $(0, 2\pi)$ by its Fourier series.

Courtesy of Mathematical Reviews R. V. Churchill, USA

2979. Mikhlin, S. G., Minimum-problem of the quadratic functional [Problema minimuma kvadratichnogo funktsionala], Moscow-Leningrad, Gosud. Izdat. Tekh.-Teor. Lit., 1952, 216 pp.

The solution of many boundary-value problems of mathematical physics can be accomplished with the aid of variational methods by solving the minimum problem of the corresponding functional. In this way, problems with linear equations can be reduced to the study of quadratic functionals.

Mikhlin's monograph is divided into four chapters. The first treats the problem from a general point of view in a complex Hilbert space. This is followed by a comprehensive explanation of the various auxiliary concepts which facilitate the application of the theoretical results to equations of elliptic type (third chapter) and to advanced problems of elasticity (fourth chapter).

Reading of the book presupposes a special knowledge of the theory of abstract spaces; however, methods under consideration represent a powerful expedient of variational calculus, and it is necessary that engineers and physicists become familiar with them. Therefore, reviewer recommends the work to all engineers who are interested in applied mathematics.

V. Vodička, Czechoslovakia

2980. Faddeeva, V. N., Computational methods of linear algebra [Vychislitel'nye metody linearnogo analiza], Moscow-Leningrad, Gosud. Izdat. Tekh.-Teor. Lit., 1950, 240 pp.

In the space of 238 pages the author presents a concise and simple exposition of the practical computational methods of linear algebra. The material is covered in three chapters. Chap. I provides basic material from linear algebra and contains a fairly comprehensive background in the following topics: Matrixes, N-dimensional vector space, linear transformations, Jordan canonical form, and the concept of limit for vectors and matrixes. A simple proof of the Hamilton-Cayley theorem is given together with properties of the minimal polynomial. The geometrical interpretation of similarity transformations is discussed with clarity.

Chap. II covers systems of linear equations. In particular, it treats Gauss's method, the evaluation of determinants, compact arrangements for the solution of nonhomogeneous linear systems, the connection of Gauss's method with the decomposition of a matrix into factors, the square-root method, the inversion of a matrix, the problem of elimination, the correction of the elements of the inverse matrix, inversion of a matrix by means of partitions into cells, method of bordering, escalator method, iteration method, preparation of a system of linear equations to a convenient form for the method of iteration, Seidel's method, and a comparison of the methods.

Chap. III is perhaps the most important one regarding applications, since it is concerned with the problem of latent (characteristic) roots. Methods are developed for obtaining both the eigenvalues and eigenvectors of algebraic systems. The systems to which the methods apply are not, in all cases, symmetric. The author develops, in sequence, the methods of A. N. Krilov, Samuelson, Danilevsky, Leverrier (with D. K. Fadeev's modification of it), the escalator method, interpolation method, and a comparison of the methods. The author, in the comparison, concludes that Danilevsky's scheme requires the least number of arithmetical operations for the entire process of computation. She also states that this method secures a comparatively high accuracy for the coefficients in the characteristic equation. She also reveals an error in H. Wayland's paper [*Quart. appl. Math.* **2**, 4] on the number of operations required for the Danilevsky method. Each method expounded is supplemented by a numerical example, borrowed from the works of A. N. Krilov and Leverrier.

All of these listed schemes yield the characteristic equation. However, for each method a control scheme for the calculations is developed. This is to secure significant figures, a feature which is important in numerical work. Further topics treat the determination of the dominant roots of a matrix, methods for improving the convergence of the iterative process, finding the subsequent roots and vectors belonging to them, matrixes with nonlinear divisors, and improving the convergence of the iterative process for solving systems of linear equations. There is a bibliography of some 36 basic papers.

The book can be used either as a text or for self study. It is particularly useful for persons concerned with high-speed digital computers. Many of the methods are also well suited for hand (desk calculator) computation, and for engineers engaged in flutter and vibration work. At the end of the text, the author gives reference to some of the more recent steepest descent or gradient methods developed by L. V. Kantorovich and other Soviet mathematicians. The text can be read and understood by persons with but little preparation. The style and presentation are clear, the ideas are simply expounded, and the proofs are easy to follow.

J. J. Brandstatter, USA

2981. **Milne, W. E., Numerical solution of differential equations,** New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd., 1953, xi + 275 pp. $6.50.

Purpose of the volume is to familiarize the applied worker with numerical techniques for solving ordinary and partial-differential equations. It is well documented with numerical examples to illustrate the discussion. The scope of the text is elementary. In this vein as much rigor as possible is given, but "rigor is not an object of idolatry." However, in contrast to a large number of engineering texts which touch on the subject, the rigor of the present book is sufficient to satisfy most applied workers. Detailed attention is devoted to errors (truncated and inherent) incurred in the numerical processes.

The volume is divided into two parts. The first deals with ordinary differential equations, while the second concerns partial-differential equations. Chap. I presents elementary notions of differential equations. A discussion of the point-slope and trapezoidal formulas is the subject of chap. II. Analytical foundations, including solutions by Taylor's series, analytic continuation, successive substitutions, etc., are considered in chap. III. Numerical integration is introduced and the chapter closes with a list of quadrature formulas useful for later work. Chap. IV contains methods based on various types of quadrature formulas. No attempt is made to present all the many methods of numerically integrating differential equations; rather, attention is given to a few that are considered most essential, and these are treated clearly and thoroughly. Chap. V takes up the method of Runge-Kutta and other techniques using higher derivatives. Systems of equations and higher-order equations are studied in chap. VI, and two-point boundary-value problems are treated in chap. VII, concluding part I of the text.

Methods of integration for partial-differential equations (part II) are of two types. In the explicit kind, numerical solution is accomplished stepwise using the given equation, initial, and boundary conditions. For the implicit type, the point values are related by a system of simultaneous equations. Chap. VIII takes up the former. Well-known parabolic and hyperbolic equations are used as examples. The chapter treats general boundary conditions, including curved, and considerable discussion is given to stencils associated with the Laplacian operator. Before discussing implicit methods and elliptic equations, author devotes chap. IX to the subject of linear equations, matrixes, latent roots and vectors, etc. The emphasis is on iterative methods. Chap. X is concerned with the elliptic equations of Laplace and Poisson. The biharmonic equation is also treated. Finally, chap. XI is concerned with the numerical determination of eigen or characteristic values of partial-differential equations.

Three appendixes are devoted to short discussions of round-off errors, large-scale computing machinery, and the Monte Carlo method. The volume is very much up to date. It has not been possible to include all methods dealing with the various topics. To provide other techniques and examples, a striking feature of the text is the inclusion of a bibliography containing over 250 items. A majority of these are publications of the last decade which attests to the modernity of this treatise.

The applied worker will find the volume an indispensable guide for the solution of differential equations. Sufficient exercises are provided so that the book is also very useful for classroom instruction. Y. L. Luke, USA

2982. **Karlqvist, O., Numerical solution of elliptic difference equations by matrix methods,** *Tellus* **4,** 4, 374–384, Nov. 1952.

Author presents a direct method for solving elliptic partial-differential equations. The idea is to split the basic matrix into triangular matrixes. Calulations of the unknown in this manner are considerably less than for calculations using the inverse matrix. It is shown that the method of solutions is stable. The technique is well suited for high-speed computers where one must solve the same equation (e.g., Poisson's) using the same network but with different right-hand rides and boundary conditions. Methods have been applied successfully on BARK, the Swedish relay computer. The ideas are also applicable to nonrectangular regions and may be applied to problems in more than two dimensions.

Y. L. Luke, USA

2983. **Jones, C. W., A short table for the Bessel functions, $I_{n+1/2}(x)$, $2/\pi K_{n+1/2}(x)$. Prepared on behalf of the Mathematical Tables Committee of the Royal Society,** Cambridge, at the University Press, 1952, 20 pp. $1.25.

In the preface to the recent, final, volume of the British Association Mathematical Tables series, Professor Bickley mentioned that the Mathematical Tables Committee of the Royal Society is proceeding with the tabulation of Bessel functions of noninteger orders. The present publication is a first installment of such tables and it is hoped that it will be followed by more extensive tables.

The principal tables of the present pamphlet give

$$x^{-n-1/2}I_{n+1/2}(x) \text{ and } 2\pi^{-1}x^{n+1/2}K_{n+1/2}(x)$$

for $x = 0(.1)5$, $n = 0(1)10$, and $e^{-x}I_{n+1/2}(x)$ and $2\pi^{-1}e^{x}K_{n+1/2}(x)$ for $x = 5(.1)10$, $n = 0(1)10$. Most values are given to seven or eight significant figures, with modified second central differences, and it is believed that the tabular values are correct to within 0.5 unit of the final decimal place. Supplementary tables give $x^{1/2}$ and $(\frac{1}{2}\pi x)^{1/2}$ for $x = 0(.1)5$, e^{-x} and $\frac{1}{2}\pi e^{-x}$ for $x = 5(.1)10$, e^{-x} for $x = 0(.01).1$, $2\pi^{-1}x^{n+1/2}K_{n+1/2}(x)$ for $x = 5(.1)6$, $n = 7(1).10$, and coefficients of the polynomials which occur in the expression of $I_{n+1/2}$ and $K_{n+1/2}$ in terms of elementary functions.

Courtesy of Mathematical Reviews A. Erdélyi, USA

2984. **Klein, H., Foundations of modern statistical methods in relation to sampling** (in German), *Arch. Eisenhüttenw.* **24,** 1/2, 11–20, Jan./Feb. 1953.

Author gives an extended discussion of the use of statistical concepts, including the chi-square and "*t*" distributions, and the use of square root and probability paper in the effective application of control methods at various confidence limits. An application is made to smelting of iron ore. Seven small numerical tables for general sampling analysis are here provided. The aim of the paper is to bring more closely together those who practice industrial sampling, as in the iron industry, and statisticians who hitherto seem to have shown more interest in biological measurements. A. A. Bennett, USA

2985. **Jahns, H., The utility of mathematical procedures in sampling, after elimination of systematic errors** (in German), *Arch. Eisenhüttenw.* **24,** 1/2, 21–26, Jan./Feb. 1953.

Author comments on the widespread reliance on empirical and traditional rules in sampling practice despite the known success of mathematical error theory in surveying and in other fields. He discusses the detection and elimination of systematic errors and the use of the mean error or of the standard deviation and the use of difference of means of lots. The paper's major contribution lies in the detailed practical discussion of error analysis in seven situations typical in the iron smelting industry, and in demonstrating how intelligent testing of the right sort at appropriate intermediate stages in the smelting process may result in impressive economies. A. A. Bennett, USA

Mechanics (Dynamics, Statics, Kinematics)

2986. Nikolai, E. L., Theoretical mechanics. I. Statics. Kinematics [Teoreticheskaya mekhanika. I. Statika. Kinematika], Moscow-Leningrad, Gosud. Izdat. Tekh.-Teor. Lit., 1951, 304 pp.

This volume is clearly designed for use as a text, the treatment beginning with simple plane concepts which are gradually extended to a complete picture (three-dimensional statics and kinematics of rigid bodies). The book is lengthy, abounding with examples and "special cases" (there are no problems included), but the unifying ideas are somewhat obscured. Some of the 266 figures are well chosen, but they are all small, the typography is crowded, and the paper is of poor quality. There is nothing in this book which is not presented as well or better in English texts. The introductory remarks on the impact of the ideas of Lenin and Stalin on theoretical mechanics in the Soviet Union are unusual.

R. E. Kronauer, USA

2987. Levi-Cività, T., and Amaldi, U., Lectures on rational mechanics. Vol. 2. Dynamics of systems with a finite number of degrees of freedom. Part II [Lezioni di Meccanica Razionale. Vol. 2. Dinamica dei sistemi con un numero finito di gradi di libertà. Parte 2], Bologna, Nicola Zanichelli, 1952, ix + 671 pp.

[For vol. I and part I of vol. II see AMR **6**, Rev. 2683.] The mode of presentation is the same as in the two earlier volumes, i.e., a rigorous treatment of a vast amount of theoretical material with applications, part of which is incorporated into problems.

The chapter headings are as follows: VII. Dynamics of rigid bodies. Some general considerations. Elementary problems; VIII. Dynamics of rigid bodies. Motion about a fixed point. Gyroscopic phenomena; IX. Dynamics of rigid bodies. Rolling motion. Systems of rigid bodies having inner cyclic motions; X. Canonical equations; XI. General principles; XII. Impulsive motion.

An index of authors quoted and an analytic index of terms employed are appended.

E. Leimanis, Canada

2988. Lemaitre, G., Symmetrical coordinates in the three-body problem (in French), *Acad. roy. Belgique, Bull. Cl. Sci.* (5) **38**, 6, 582–592, 1952.

Author considers first the three-body problem in the plane and with equal masses. Taking the origin of the coordinate system in the mass center and the (x, y) axes along the principal axes of inertia, he introduces three parameters r, σ, and χ such that $x_i = ar \cos \sigma \cos \chi_i$; $y_i = ar \sin \sigma \sin \chi_i$; $a = (2/3)^{1/2}$; $\chi_i = \chi + (2/3)\pi i$, $i = 1, 2, 3$. A fourth parameter ψ is the angle of the x-axis and a fixed line in the plane. Author derives the Hamiltonian and shows that the motion of the system corresponds with that of a single particle under certain conditions. The particular problem is taken as a starting point for the study of the general problem, the Hamiltonian of which is derived also.

O. Bottema, Holland

2989. Avilov-Karnaukhov, B. N., An investigation of mechanisms by energy characteristics (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **84**, 1, 25–28, May 1952.

Author proposes to use the energy characteristics for investigation of mechanisms. These characteristics are the bounds which represent the qualities of mechanisms and the conditions of their work. These bounds are expressed not only by certain indexes, but also by a certain number of secondary factors. Therefore, author assumes that the energy characteristics represent the statistical bounds.

These considerations are illustrated by the example of the movement of coal in a conveyer. Author describes the energy characteristics of this mechanism as a linear equation and defines the parameters of such a characteristic, using the notions and the laws of mathematical statistics. He obtains the mean and probable values of the frictional resistance coefficient of the conveyer w', which changes within the limits from 0.5 to 0.6, and also the mean values of the frictional coefficient between the chain and the channel of the conveyer.

W. Wierzbicki, Poland

2990. Shavliuga, N. I., Kinematic chains of metal-cutting machinery [Kinematicheskiye tsepi metallorezhushchikh stankov], Moscow-Leningrad, Gosud. Nauch.-Tekh. Izd. Mash. Lit., 1950, 288 pp.

Author assumes that the reader is familiar with general terminology and descriptions of machines. Kinematic chain of a metal-cutting machine is defined by author as a system of any links and components of the machine so connected as to transfer motion from the drive source to other parts and from one component to another.

A system of kinematic chains which are connected among themselves are represented by author schematically by diagrams which are called "structure formulas." These diagrams appear to be a convenient form of representing functional features of a machine. Kinematic chain may consist of groups of interchanging and permanent elements. Such a group is identified as a "section" of a kinematic chain. A group of interchangeable elements is identified as a "structural section" or "buildup section." Each section has a numerical characteristic which in general is equal to the product of numerical characteristics of kinematic pairs within such a section. Numerical characteristic of a kinematic chain in general also is a product of numerical characteristics of kinematic pairs and sections which compose the chain. For a kinematic chain consisting of gears only, its numerical characteristic will be the over-all gear ratio. Numerical characteristics of a kinematic chain determines relationship between its end elements. When displacement of one element is assigned, then so-called design displacement remains unknown. Equations governing displacements of end elements are called equations of the kinematic chain. Unknowns in such equations are numerical characteristics of structural sections. Solution of these equations are known as design formulas. These formulas provide relationship between unknown numerical characteristic of a structural section and the required design displacement of end element. Equations may be simple linear or quadratic, and, in complex cases, differential equations may be necessary. The above definitions and propositions form a foundation for author's analysis of this subject. Considerable credit is given by author to academician A. V. Gadolin for introducing in 1877 a mathematical foundation for effective arrangements of step-sets of the number of revolutions per minute for a spindle of a machine. These step-sets follow a geometric progression. Author confines his work to machines predominantly designed in his country. The following types are analyzed: Engine and turret lathe; universal indexing, dividing heads; universal and vertical milling machines; vertical and horizontal boring and drilling machines; shapers and planers; semi-automatic toothed wheel milling (punching, shaping); semi-automats for cutting screw and toothed wheels (conical, normal, and conical curvilinear); screw and toothed-wheel polishing machines.

Reviewer feels that this text may be of some interest to designers who are particularly interested in various approaches to the analysis.

V. A. Valey, USA

2991. **Hughes, J. V., Possible motions of a sphere suspended on a string (the simple pendulum),** *Amer. J. Phys.* **21,** 1, 47–50, Jan. 1953.

Author discusses the motions of a specific compound pendulum. Simple analysis yields the frequency equation. The low-frequency solution is shown to represent a motion very close to that of a simple pendulum. The high-frequency result represents a harmonic rolling of the sphere about an axis near its center with little attendant angular motion of the supporting string. Photographs are presented illustrating the two motions.

Reviewer agrees with author that results given in this paper are explicit details derived from well-known theory rather than fundamental contributions. R. B. Grant, USA

2992. **Gambo, K., and Kasahara, A., Notes on the motion of a rotating solid cylinder on a rotating earth,** *J. meteor. Soc. Japan* 2) **30,** 4, 119–125, Apr. 1952.

The complete equation of motion for the cylinder shows that a "new force" contributing to its motion suggested by Syōno does not exist and that his result was the product of an incorrect coordinate transformation. J. C. Freeman, USA

Gyroscopics, Governors, Servos

2993. **Poritsky, H., Topics in gyroscopic motion,** *J. appl. Mech.* **20,** 1, 1–8, Mar. 1953.

Using unit sphere analysis, the motion of an ideal gyro, with perfect alignment and balance and with inertia concentrated in a rotor which is a solid of revolution, is considered for (a) zero torques and precessional torques applied to gimbals, and (b) torque applied to rotor via gyromotor. Effects in case (a) of gimbal inertias and of departure of rotor from solid to revolution are then considered. L. Becker, USA

Vibrations, Balancing

(See also Revs. 2975, 2991, 3178)

2994. **Thompson, S. P., Statistical mechanics of complex vibrating systems,** Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 49–56, 1952.

Author considers vibrations in which properties of the systems are known but in which excitations are governed by probability distributions. He employs concept of a hypothetical ensemble of identical independent systems, each like the system under study. The probability that the original system at a given time is in a particular state of motion—i.e., that it may be represented by a particular point in phase space—is proportional to the number of systems in the ensemble which are in the same state at that time. Small vibrations of linear conservative systems are chosen as an example. Initial conditions are assigned on basis of equal probability, and solutions are given for probabilistic distributions of mode energies and internal deflections of the system.

W. C. Hurty, USA

2995. **Ku, Y. H., Nonlinear analysis of electro-mechanical problems,** *J. Franklin Inst.* **225,** 1, 9–32, Jan. 1953.

Author considers the differential equation $\ddot{x} = g(x, \dot{x})$ which, by use of $\dot{x} = v$, may be given the form $dv/dx = g(x, v)/v$, and he wishes to integrate this differential equation in the phase plane ($v - x$-plane) by graphical means. He recommends drawing a family of curves $y = g(x, v_a)$ for a set of values v_a, and points out that the tangent to the trajectory in the phase plane at point x_1, v_1), having slope $g(x_1, v_1)/v_1$, is perpendicular to the hypotenuse of the right triangle having v_1 as its vertical side and $g(x_1, v_1)$ as its horizontal side (drawn to the right of x_1 if $g > 0$, to the left if $g < 0$).

This procedure, then, is used for integrating the two forms of van der Pol's differential equation, the differential for the linearly damped pendulum, and the differential equation where

$$g = -k(1 - b \cos 2x)v - r \sin 2x - \sin x$$

If the differential equation contains a forcing function $F(t)$ in addition to $g(x, v)$, the suggestion is that this function $F(t)$ be treated as stepwise constant.

Recently, a number of papers, besides the present one, have dealt with graphical constructions in the phase plane. Two of them merit mention; the short note by M. Schäfer [*ZAMM* **32,** 8/9, 285–286, Aug./Sept. 1952] which has some traits in common with the present paper, and the paper by L. S. Jacobsen [AMR **6,** Rev. 2124] in which also the suggestion is offered of treating a forcing function as stepwise constant. K. Klotter, USA

2996. **Klotter, K., Non-linear vibration problems treated by the averaging method of W. Ritz,** Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 125–131, 1952.

Paper presents a mathematical method for studying the steady-state vibrations of dynamical systems whose equations of motion are nonlinear. The method is similar in principle to the Ritz-Galerkin method for solving boundary problems in statics of elastic systems.

The method is described and then applied to the solution of typical conservative and nonconservative nonlinear dynamical systems having one degree of freedom. "One-term" approximations that satisfy certain "averaging conditions" are obtained. In principle, the method presented is very similar to that given by G. Schwesinger [AMR **4,** Rev. 1465] and is a valuable addition to the list of known methods for approximating the solutions of nonlinear vibration problems. L. A. Pipes, USA

2997. **Goodman, L. E., Circular-crested vibrations of an elastic solid bounded by two parallel planes,** Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 65–73, 1952.

Two fundamental radially symmetric solutions of the linear equations of elasticity are found to describe free vibrations of an elastic plate of finite thickness. It is shown that they correspond to "extensional" and "flexural" motions and are analogous to the straight-crested (plane strain) waves discovered by Lamb and Rayleigh. At large distances from the origin, the circular-crested and straight-crested motions become indistinguishable.

Following the treatment of Lamb, the transcendental equation relating the frequencies of vibration to the wave shape is developed. A numerical tabulation of the solutions of the frequency equation is given and the shapes of the modes are described in detail. For a particular set of modes there exist surfaces across which no stress is transmitted. These surfaces define shapes for which the motion in question represents a mode of free vibration.

In the limit, for very thick plates, the solutions are shown to correspond to Rayleigh surface waves while, for thin plates, they approach asymptotically well-known results in elementary plate theory. W. Kochanowsky, Germany

2998. **Anderson, R. A., Flexural vibrations in uniform beams according to the Timoshenko theory,** ASME Appl. mech. Div., Semi-Annual Meet., Los Angeles, Calif., June 28–July 2, 1953. Paper 53—SA-9, 7 pp.

Author deduces the transverse response of uniform beams in

accordance with the Timoshenko bending mechanisms. Various sets of boundary conditions are included in the considerations. The general method employed for obtaining mathematical solutions is based on the classical principle of mode superposition. Solutions appear as power series expansions which do have certain numerical advantages over the exact solutions obtained by Dengler and Goland [AMR 6, Rev. 2703]; while, on the other hand, it might be generally admitted that the exact traveling wave approach conveys a deeper insight into the physical reality of the respective phenomena.

Contents of paper are closely related to the analysis of J. Miklowitz [title source, Paper 53—SA-6]. Results of all three investigations, the work of the present paper and the works reported in references cited, are in excellent agreement. The idea to treat rotatory and shear deflections separately instead of dealing directly with the Timoshenko differential equation in the total transverse displacement is a favorable one. It simplifies the establishment of boundary conditions.

M. A. Dengler, USA

2999. Miller, D. F., Forced lateral vibration of beams on damped flexible end supports, *J. appl. Mech.* **20,** 2, 167–172, June 1953.

Author gives details of an iteration method for solving lateral vibration equations of a single-span beam mounted on damped elastic supports. Numerical results are given for two examples.

W. H. Hoppmann, II, USA

3000. Smelkov, A. A., Transverse vibration of the drum of a centrifuge of the "Tubulyar" type (in Russian), *Inzhener. Sbornik, Akad. Nauk SSSR* **9,** 57–64, 1951.

Article deals with the vibrations of the drum of a centrifuge caused by statical unbalance. The long thin drum is considered as a homogeneous elastic bar. In the case treated, the center of gravity of each section perpendicular to the axis is assumed to have a constant distance ρ to the axis; moreover, it is supposed that all these centers lie on a straight line, this being the worst unbalance the drum can have. It is further assumed that the supports can take only lateral forces and no moments.

The differential equations of the problem are derived from Hamilton's principle. These are two linear simultaneous partial-differential equations of the fourth order in the displacements x and y in two mutual perpendicular directions, with constant coefficients and periodical terms originating from the unbalance in the second members. These equations are combined to one equation in the complex variable $u = x + iy$. From this equation, the critical angular velocities of the unbalanced drum and the natural frequencies of the balanced drum ($\rho = 0$) are calculated.

W. H. Muller, Holland

3001. Harker, R. J., Vibratory response of rigid rotors to dynamic unbalance, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 119–124, 1952.

Author treats excitations from unbalance forces of rigid rotor in conventional balancing machine with two degrees of freedom. Neglecting damping, general expressions for amplitudes at bearings and for two conventional methods of unbalance separation are developed. New method of unbalance separation is also proposed utilizing two particular speeds of rotation, at each of which the amplitude of one bearing only depends on one of the unbalance components. (In formulas [16] and [17] determining these speeds, the left-hand side should be squared.) Paper concludes with comparison of available amplitudes for the three methods of separation, but it is pointed out that this is not the only criterion for accurate unbalance determination. For practical viewpoints of unbalance separation, see AMR 2, Rev. 972.

S. Sjöström, Sweden

3002. Fedorovich, V. N., An acoustic method for vibration measurement (in Russian), *Zh. tekh. Fiz.* **22,** 2, 238–244, Feb. 1952.

A method is described for overcoming difficulties which arise when a uniform frequency characteristic is to be determined for frequencies from 100 to 5000–6000 cps.

From author's summary by W. S. Jardetzky, USA

3003. Prang, G., Educational and experimental models for the representation of linear, circular, and elliptic vibrations (in German), *Feingerätetech.* **2,** 1, 11–15, Jan. 1953.

Instruments to project mentioned vibrations on a projection screen in a lecture room are discussed and described. An arrangement consisting of two perpendicular crossed bars, each making a harmonic movement in a plane perpendicular to the beam of light, is recommended. As the point of intersection of the two bars intercepts only half as much light as the two bars, this point is presented on the screen as a brighter spot on a darkened background. By changing the phase angle between the two movements, linear, circular, and elliptic vibrations can be obtained.

R. G. Boiten, Holland

Wave Motion, Impact

(See also Revs. 2998, 3012, 3107, 3134, 3233, 3235, 3242, 3248, 3269, 3270, 3273, 3282)

3004. Pursey, H., The motional impedance of simple sources in an isotropic solid, *Brit. J. appl. Phys.* **4,** 1, 12–20, Jan. 1953.

Apparently by analogy with the electrical case, ". . . the motional impedance is obtained by dividing force by velocity of source." Intended as a first step toward analyzing more complex sources such as a piston on the free surface of a semi-infinite medium. Author uses the classical equations of wave propagation to give particle displacement and hence, by differentiation, the strains and so the stresses. Cases considered are dilating cylinder and sphere, and oscillating cylinder and sphere.

K. H. Swainger, England

3005. Haskell, N. A., The dispersion of surface waves on multilayered media, *Bull. seism. Soc. Amer.* **43,** 1, 17–34, Jan. 1953.

Author discusses propagation of plane waves in a multilayered medium. Employing a matrix formalism for expressions for displacements, stresses, and boundary conditions in the medium, an arranged form of dispersion function of phase velocities is obtained which is so general that it can describe characters of Rayleigh wave, Stoneley wave, and others according to assumptions on structures of layered medium. Love wave and fluid coupled wave are also discussed.

Numerical examples are shown of dispersion of Rayleigh wave in two- or three-layered medium, and comparison is made with seismological data.

The reviewer suggests that the author discuss the amplitude- or energy-division of incident waves in multilayered systems, as T. Matsumoto has started.

K. Kasahara, Japan

3006. Mindlin, R. D., and Herrmann, G., A one-dimensional theory of compressional waves in an elastic rod, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 187–191, 1952.

See AMR 5, Rev. 1308.

3007. **Cicala, P., On the determination of the performance characteristics of a tire-oleo landing gear** (in Italian), *Monogr. Lab. Aero. Politec. Torino* no. 291, 11 pp., 1952 = *Atti Accad. Sci. Torino* **86**, 1951–1952.

Paper gives a method for stepwise numerical integration of the equation of vertical linear motion of an airplane landing on its elastic tire with an oleo shock absorber in between. The force-displacement characteristic of the tire and the force-velocity-displacement function of the shock absorber are both decidedly nonlinear. Inertia effects of the landing gear are neglected with respect to the inertia of the airplane itself.

J. P. den Hartog, USA

3008. **Hazard, R. F., Undercarriage performance during landing,** *Aircr. Engng.* **25**, 287, 16–19, Jan. 1953.

Report presents a mathematical solution for determining maximum undercarriage loads and shock-absorber orifice area required for a landing gear of given oleo stroke and tire constants. The method involves setting up force and momentum equations and reducing complexity by restricting certain boundary conditions. Approximations are discussed and indicate the approach for applying purely mathematical analyses to industrial-type problems. Solution is given implicitly in curves derived explicitly giving functions of maximum load $F(t)$ in terms of its static and dynamic components. Results are stated to agree well with experiment. Not considered in analysis are free body pitch and roll of the airplane and wheel spin-up. Author's definition of "tire stiffness" should be inverted for proper use in equations.

K. Arnstein, USA

Elasticity Theory

(See also Revs. 2976, 3004, 3006, 3037, 3068)

3009. **Hlitčijev, J., and Vrečko, M., Strength of materials (Otpornost Materijala),** Belgrad, Izd. Preduz. Narod. Repub. Srbije, 1952, vii + 295 pp.

As stated by the authors, this book is intended primarily as a textbook for undergraduate students. It seems to be well suited for this purpose. The bending of beams and the buckling of bars are treated according to classical methods and are the main topics. A short chapter deals with the bending of circular plates.

No original methods are presented. Simple problems are included at the end of each chapter, which aid in giving purpose to the underlying principles and theory presented.

M. M. Stanišić, USA

3010. **Filonenko-Boroditsch, M. M., Strength of materials.** vol. II [**Festigkeitslehre, Bd. II**], Berlin, Verlag Technik, 1952, 511 pp. $2.28.

This volume on strength of materials is the second of a two-volume set. Just as for volume I [AMR **6**, Rev. 376], it is the contribution of five Russian authors, but now I. N. Kudrjawzew is replaced by I. W. Urban as one of the authors.

The first half of the book is devoted to more advanced questions on the bending of beams and the theory of indeterminate frames. It includes treatment of large deflection of elastics, influence of shear on the shape of elastic line, nonlinear stress-strain relations, curved beam, influence of longitudinal loads on bending, and beam on elastic foundation.

Analyses are made by means of differential equations, principle of virtual work, Ritz-Timoshenko series method, energy method, and Castigliano principle. Typical of the methodology of the book is the presentation of Kryloff's modification of the Cauchy method for solving complete, nonhomogeneous, fourth-order, ordinary differential equations for use with beam theory.

The second half of the book contains rather brief treatments of the bending of plates, membrane stresses in containers, twisting of bars, twisting and bending of thin-walled beams, elastic stability of ring, sidewise buckling of beam, buckling of tube, and buckling of plate.

Chapters are devoted to stress measurements, stress concentration, strength theories, plasticity, and mechanical properties of materials. The chapter on plasticity is extremely brief. In the chapter on measurements, the theory of the resistance-wire strain gage is presented, with no mention of SR-4. Photoelasticity is also treated in this chapter, with most of the effort concentrated on the analytical determination of the isoclinics, isochromatics, and isopachics. The Lamé-Maxwell equations are derived and used. In the chapter on properties of materials, fatigue, impact, creep, and cold-work are discussed.

The format and print are the same as for vol. I and are good. The book is devoted entirely to statics, except for the standard impact tests. About 30 problems, mostly with solutions, are scattered throughout.

The reviewer considers the two volumes to be an excellent treatment of the subject. An English translation would be of considerable value for use in this country. The greatest defect in the work is the paucity of references so well presented in the renowned books of S. P. Timoshenko. W. H. Hoppman, II, USA

3011. **Pini, B., On the boundary-value problem of the first kind of the theory of elasticity** (in Italian), *R. C. Semin. mat. Univ. Padova* **21**, part 2, 345–369, 1952.

Existence of solution for the first problem of the theory of elasticity (i. e., when displacements at the boundary are given) was proved many years ago by Lauricella (1906), and afterward by Marcolongo, Korn, Boggio (using integral equations). Problem was resumed by Lichtenstein (1924) and solved with only an integral equation by Friedrichs (1947) who used Courant's methods of calculus of variation, and by Fichera (1948) who used equations of Fischer-Riesz as considered by Picone.

Author now solves the same problem, using a method given by G. Cimmino for partial-differential equations. Solution does not require any knowledge of integral equation but is obtained by successive approximations. G. Supino, Italy

3012. **Ericksen, J. L., On the propagation of waves in isotropic incompressible perfectly elastic materials,** *J. rational Mech. Analysis* **2**, 2, 329–337, 1953.

Author treats the problem, using Truesdell's tensor notation, and derives the relations for higher-order waves. He applies these relations to large deformations of perfectly elastic materials which obey the Mooney-Rivlin conditions.

H. Kolsky, England

3013. **Sáenz, A. W., Uniformly moving dislocations in anisotropic media,** *J. rational Mech. Analysis* **2**, 1, 83–98, 1953.

Author considers a closed loop of dislocation of general type traveling with uniform velocity through a homogeneous anisotropic medium. It is shown that a steady-state solution exists provided the speed does not exceed a certain limiting value which depends on the direction of motion, specified by the unit vector λ. For a given λ there are three velocities of wave propagation. The critical speed of a dislocation always lies between the greatest and least values of the smallest of these three velocities, considered as a function of λ. In an isotropic medium these extreme values coincide with the speed of transverse waves a.

The displacement field of a uniformly moving loop is expressed in terms of a function ${}_su_i^*(\mathbf{x}, \mathbf{x}_0)$. Here $\mathbf{x}$ and $\mathbf{x}_0$ are co-ordinates in a frame moving with the dislocation, and ${}_su_i^*$ is the ith component of the displacement at $\mathbf{x}$ due to a unit force in the direction x_0, acting at the (moving) point $\mathbf{x}_0$. The existence of this function is deduced from a theorem of Fredholm, but its explicit form is not given.

The results of Weingarten and Volterra are generalized to the case of uniformly moving dislocations.

F. R. N. Nabarro, England

3014. Olszak, W., Nonlinear torsion of anisotropic bars (in Polish), *Arch. mech. stos.* **3,** 3/4, 225–257, 1952.

Article has eight parts. Author gives, first of all, a formulation of his problems and sketches some basic equations of nonlinear theory of elasticity. Third section gives (in analogy with the well-known considerations of C. Weber for the case of isotropic material) a simplified theory of nonlinear torsion of anisotropic bars, especially for rectangular cross section. Then comes a theoretical discussion of secondary longitudinal stresses due to great distortion and of their influence in increasing torsional rigidity of the bar.

In the sixth part, author summarizes his theoretical results and indicates the most substantial differences between the nonlinear torsion of isotropic and anisotropic bars. Numerical examples illustrate preceding theory.

Paper is evidently of technical value. Mathematical deductions are very simple and can easily be followed by engineers.

V. Vodička, Czechoslovakia

3015. Nelson, C. W., Ancker, C. J., Jr., and Wu, N.-G., The stresses in a flat curved bar due to concentrated radial loads, *J. appl. Mech.* **19,** 4, 529–536, Dec. 1952.

Fourier integral solution is given for problem of curved bar bounded by two concentric circles and loaded in plane of curvature by combination of radial loads on circular boundaries. Numerical results are presented for a curved cantilever subjected to radial concentrated load on outer surface directed toward center of curvature. An example is included to show how superposition will obtain stress for loading frequently found in machine-design problems.

D. E. Hardenbergh, USA

3016. Okubo, H., On the position of the axis of twist, *Rep. Inst. high Speed Mech., Tōhoku Univ.* **1,** 87–92, 1951.

Expression is obtained for relation between twisting moment acting on a shaft, angle of twist per unit length, location of the axis of twist, and length of shaft. Notation is that of Love; references (to 1934 only) are incomplete; translation is inadequate.

E. W. Suppiger, USA

3017. Brush, D. O., and Sidebottom, O. M., Axial tension and bending interaction curves for members loaded inelastically, *Trans. ASME* **75,** 1, 63–71, Jan. 1953.

See AMR **6,** Rev. 1194.

3018. Horvay, G., The end problem of rectangular strips, *J. appl. Mech.* **20,** 1, 87–94, Mar. 1953.

Author studies stress in a semi-infinite rectangular strip on the ground of the variational method, used in the manner of Ritz, i.e., satisfying the boundary conditions exactly but the stress equations only approximately. The results show the decrement of stress when self-equilibrating forces are applied on the finite side of the strip only. Applications will follow in subsequent papers.

G. Supino, Italy

3019. Berger, E. R., Torsional rigidity and shear center of a multicellular hollow plate with equidistant webs (in German), *Öst. Bauzeitschr.* **8,** 1, 7–11, Jan. 1953.

The plate investigated is actually a beam, because two-dimensional effects are not investigated at all. According to the title, paper is divided in two parts:

(1) The author applies Marguerre's theory for torsion of multicellular beams [*Bauingenieur,* 317–322, 1940]. He obtains a difference equation for the shear flows in the cells of the section; integration then gives a relatively simple explicit expression for the torsional rigidity. Some approximate but simpler formulas are then derived. A numerical example is given.

(2) Horizontal plates composing the beam are supposed to be unequally thick, as in some cellular bridges, so that the section has no horizontal axis of symmetry. The ordinate of the shear center is investigated as follows: The beam is subjected to bending with shear with the vertical as neutral axis; then, a horizontal cut is made across all the webs. The shear stress distribution is calculated for each of the two parts so obtained. Statically indeterminate shear flows are introduced in each cell for establishing the continuity of the longitudinal displacement at the cuts. These flows obey a difference equation which is integrated. Finally, the shear center is obtained as the point of application of the resultant of interior forces. Formula obtained is very complicated, but nevertheless usable.

A cellular concrete bridge section is offered as a numerical example. Paper is very clearly and compactly written.

Ch. Massonet, Belgium

3020. Horvay, G., Thermal stresses in perforated plates, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 247–257, 1952.

Author develops method of stress analysis for a plate perforated by circular holes, the centers of three adjacent holes forming an equilateral triangle. Using elementary beam theory, he works out displacements of characteristic element of plate under tension, shear, and bending. These results are used to analyze the whole plate when loaded in a few simple ways. Author's statement, that because the over-all elastic constants are the same in two perpendicular directions the plate is isotropic, is not correct. It is a necessary but not sufficient condition.

D. R. Bland, England

3021. Parkus, H., Thermal stresses in axially symmetrical shells in case of axially symmetrical temperature distributions (in German), *Öst. Akad. Wiss. math.-nat. Kl. S.B.* IIa **160,** 1–5, 1–13, 1951.

Starting from Meissner's basic equations for rotational shells, author derives additional terms for computing thermal stresses due to symmetric distribution of temperature. Special solutions for spherical, conical, and cylindrical shells are developed.

H. Neuber, Germany

3022. Conway, H. D., Some problems of orthotropic plane stresses, *J. appl. Mech.* **20,** 1, 72–76, Mar. 1953.

Many plane-stress problems in isotropic elasticity are solved conveniently by considering the Airy stress functions in the form of Fourier series (periodic loading) or Fourier integrals (nonperiodic loading) of harmonic and biharmonic functions. The object of this paper is to show how many of these solutions can be extended to the case where the material is orthotropic (orthogonally anisotropic). For illustrative purposes, the following seven problems are solved: (a) Half plane subjected to normal loading; (b) wedge subjected to axial loading; (c) half plane sub-

jected to a normal force applied through a rigid punch; (d) half plane subjected to tangential loading; (e) wedge subjected to transverse loading; (f) force applied at a point in an infinite plane; (g) analysis of deep beams.

From author's summary by J. H. Baltrukonis, USA

3023. **Conway, H. D., The stress distributions induced by concentrated loads acting in isotropic and orthotropic half planes,** *J. appl. Mech.* **20,** 1, 82–86, Mar. 1953.

Paper is continuation of work begun earlier (see preceding review). Use is made here of the solution found therein for a single concentrated load acting on an orthotropic media, as is done by Timoshenko for isotropic media. Stress function for concentrated load acting on half plane is represented by a Fourier integral in the manner of K. Girkmann ("Flächentragwerke," Julius Springer-Verlag, Vienna, 1946). Use is made of solution of whole plane problem with image load placed symmetrically in the reflected half plane to find necessary loading along straight edge. Problems solved are the isotropic and orthotropic, half plane with load line normal to straight edge, and orthotropic half plane with load line parallel to straight edge. Author observes that orthotropic results reduce to known isotropic solutions when the moduli of elasticity in the directions of orthotropy are equal.

H. Cohen, Israel

3024. **Pruzsinszky, J., Stress calculation of piping lying in a plane and fixed at both ends** (in Hungarian), *Magyar Energiagazdaság* **4,** 4, 115–122, 14 figs., Apr. 1951.

The stress calculation of piping lying in a plane and fixed at both ends must be based on the deformation caused by the increase of temperature. Simultaneously, it should be taken into account that frequently the end of the piping also suffers displacement, e.g., as in the case of connections to steam turbines or to boilers. The elasticity of the piping is influenced by its shape, the developed length of the piping system, the dimensions of the pipe, and by Young's modulus of elasticity. It also depends on the temperature of the piping material. From the mathematical expression of elasticity, by applying Castigliano's theorem, it is possible to derive the restoring force, i.e., its two components which must pass through the center of gravity of the system. By applying this, as well as the von Kármán principle, it is possible to calculate the stresses resulting from the thermal expansion of the piping system and the resultant stress. Instead of lengthy theoretical calculations, the graphical tables worked out by Schwedler and Jürgensonn can be used with advantage for simple pipe lines. The determination of the stresses resulting from heat expansion is a lengthy process, requiring patience and precision.

Courtesy of Hungarian Technical Abstracts

Experimental Stress Analysis

(See also Rev. 3059)

3025. **Fessler, H., and Rose, R. T., Photoelastic determination of stresses in a cylindrical shell,** *Brit. J. appl. Phys.* **4,** 3, 76–79, Mar. 1953.

The frozen-stress photoelastic technique is used to determine the surface stresses and deformations of a cylinder with flat head closure (2.7-in. bore, 0.380-in. wall thickness) under internal pressure (7.4 psi). The material used was Marco Resin SB.26.C; the technique of casting, machining, freezing, and slicing of the models is described. The observed stresses and deformations are compared with values calculated from an analysis by Watts and Burrows [AMR 2, Rev. 591] and found to be in fair agreement except near junction of head and shell.

From authors' summary

This discrepancy, which should not be allowed to overshadow the general confirmation of the elastic theory of pressure-vessel analysis, is attributed by the authors to assumptions embodied in thin-plate theory (diameter/wall-thickness ratio = 8.15 in this case). Reviewer suggests that the presence of a rounded fillet at the interior of the juncture of the model head and body may also account for some of the effect observed. For details of the analysis of this type of pressure-vessel closure, engineers may also refer to the work of Watts and Lang [AMR **5,** Rev. 1063].

L. E. Goodman, USA

3026. **Tsien, H. S., Similarity laws for stressing heated wings,** *J. aero. Sci.* **20,** 1, 1–11, Jan. 1953.

Paper develops mathematical equations for the deformation of an unequally heated plate. These are compared with the corresponding equations for a loaded plate at uniform temperature, and conditions are formulated which would insure similarity of deformation in the two cases. The basis is thus laid for an analog machine for temperature stressing of solid and box wings. However, the required experimental setup is complex, involving loading at many points and "feed-back" (some loads depend upon the deformation). Reviewer feels that effort to overcome the difficulties of direct testing might be more profitable than the use of this analog.

W. S. Hemp, England

3027. **Makowski, Z. S., and Pippard, A. J. S., Experimental analysis of space structures, with particular reference to braced domes; with a note on stresses in supporting ring-girders,** *Proc. Instn. civ. Engrs.*, part III, **1,** 3, 420–441, Dec. 1952.

Maxwell's theorem of reciprocal deflections was used as the basis of analyzing experimentally a simple braced dome and a model of the Dome of Discovery (built for the 1951 Festival of Britain). The simple dome model was made from $^1/_8$-in. wire with welded joints. Influence coefficients for reactions were found by inducing three mutually perpendicular displacements on the foot of supporting strut, and measuring nodal displacements with cathetometers. Criterion for elastic similarity is discussed.

Model of Dome of Discovery, made of mild steel ribs and ring girder and brass tubes for bipods, was analyzed by same method. Turnbuckles in several members were used to change length of members to find influence coefficients of stress for each member. Relaxation method was used on both models for theoretical analysis. Formulas given for ring girder are checked separately by comparing measured and calculated displacements at points around a model ring.

Reviewer believes agreement between experimental and theoretical results is excellent and unusual in structures of this complexity.

T. Grow, USA

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 3009, 3014)

3028. **Dietrich, G., Calculation of spur and parallel helical gears. Testing of gear materials. Gear tests [Berechnung von Stirnrädern mit geraden und schrägen Zähnen. Prüfung von Zahnrad-Werkstoffen. Zahnradversuche],** Düsseldorf, Deutscher Ingen.-Verlag, 1952, xii + 127 pp. DM 16.80.

This book is a welcome contribution to the extensive existing literature on gears. Author, who is known for his research work on friction and noise of gears, has written this volume from the gear designer's point of view.

Chapters A, B, and C deal with basic gear calculations. An unusually extensive treatment of the calculation of parallel helical gears is given in chap. D. Chapters E, F, G, and H cover internal

gears, backlash, and the calculation of the most important inspection dimensions. Chap. J is denoted by the calculation of the bending strength, contact strength, heat at the tooth face due to friction, and resistance to wear. In the subsequent chap. K, test methods for gear materials and gears are discussed critically. In order to make the calculation effort of the gear designer easier, author has developed schemes for the calculation of gear dimensions. The determination of these dimensions (especially for severe conditions) normally requires repeated calculations due to modified assumptions. This can be overcome by the indicated schemes without sacrificing accuracy of the result, as is usually true with simplifying approximation methods. Furthermore, a considerable reduction of calculation is gained by applying a graphical system of seven tables (chap. M). The preparation of these tables is based on the assumption of theoretical equal strength at the tooth roots of meshing pinion and gear. The concluding chap. N treats three typical examples for illustration of author's new method of gear calculation.

Reviewer believes that the introductory chap. A would gain by explaining kinematically the law of conjugate gear-tooth action by means of the theorem of the three instantaneous centers of three planes having plane relative motion.

The book is clear and concisely styled. It contains a complete system of symbols covering each value used in the text. The presented calculation schemes with the tables seem to be very useful to designers who only occasionally deal with gear trains. To a certain extent, the author has given the results of this research activity and experience with the well-known "Zahnradfabrik-Friedrichshafen." J. Boehm, USA

3029. Grünert, S., On the calculation of flank pressures of "V-type" spur gear systems (in German), *Maschinenbautech.* **1**, 8, 361–364, Nov. 1952.

Paper presents tables for the calculation of flank pressures in "V-type" spur gear systems. While they might prove useful for the designer, their theoretical foundations do not seem to be absolutely correct. L. Föppl, Germany

Plates, Disks, Shells, Membranes

(See also Revs. 3020, 3025)

3030. Dean, W. R., The Green's function of an elastic plate, *Proc. Camb. phil. Soc.* **49,** part 2, 319–326, Apr. 1953.

Author considers the problem of finding the deflection of a thin elastic plate of infinite extent, clamped along two collinear, semi-infinite lines, and subjected to a transverse force applied at an arbitrary point of the plate. By mapping conformally the area of the plate on a unit circle, he reduces the problem to one of a circular plate for which he is able to find the deflection function in closed form. Some numerical results are included in the paper.
H. Deresiewicz, USA

3031. Chenea, P. F., and Naghdi, P. M., Graphical analysis of axially symmetrical plates with variable thickness, *J. appl. Mech.* **19,** 4, 561–563, Dec. 1952.

Finite difference methods and graphical integration are used to obtain numerical values for the deflection in an axially symmetrical plate with variable thickness. For a uniformly loaded circular plate the results are found to be in good agreement with those already known. B. R. Seth, India

3032. Reissner, E., Pure bending and twisting of thin skewed plates, *Quart. appl. Math.* **10,** 4, 395–397, Jan. 1953.

In the elementary theory of thin rectangular plates, two cases of pure bending arise when uniformly distributed bending moments M_1 and M_2 are applied to opposite pairs of edges, and also when oppositely directed pairs of loads are applied at diagonal corners of the plate. Author provides similar cases of pure bending and torsion of thin skewed (parallelogram) plates and gives the effect of this skewness on the bending and torsional rigidity of plate. In the case of torsion of skew plates the deflection w is given by

$$w(x, y) = k_1 [xy - k_2(y^2 - \sigma x^2)]$$

where k_2 involves the angle of skewness and vanishes for rectangular plates, and k_1 is the same constant for both types of plates. An immediate result is that a skew plate has a smaller torsional rigidity than a nonskew plate. In pure bending the deflection is

$$w(x, y) = k_3 [x^2 - \sigma y^2 - k_4 xy]$$

where k_3 is the same constant for both types, and k_4 vanishes in rectangular plates. The constants k_2, k_4 involve the angle of skewness, and k_1, k_3 are independent of it. Skew plates have a smaller bending stiffness than nonskew plates and at the same time are accompanied by a torsional deformation. A leading factor 1/2 is missing from the last equation of the paper.
D. L. Holl, USA

3033. Pell, W. H., and Prager, W., Limit design of plates, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 547–550, 1952.

Prandtl-Reuss material of perfect plasticity with incompressible plastic flow is assumed, with octahedral shear stress as the criterion for plastic flow. The lower bound of the limit load is taken as the one stressing the plate at some point up to the yield octahedral shearing stress. The upper bound of the limit load is found by equating the rate of external work to the rate of internal energy dissipation due to plastic flow in the whole plate, with velocity fields satisfying the boundary conditions of the plate. The smallest load with a velocity field satisfying the boundary conditions is the limit load. An example is given showing the determining of the limit load of a simply supported circular plate under uniform load to within 5.3%. T. H. Lin, USA

3034. Pickett, G., and McCormick, F. J., Circular and rectangular plates under lateral load and supported by an elastic solid foundation, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 331–338, 1952.

General expressions are derived in polar co-ordinates for the axial displacement w, due to a surface pressure p on an elastic layer of thickness t, by means of infinite integrals involving Bessel functions of the radial co-ordinate r. Analogous general expressions are constructed as solutions of the circular plate with sufficient parameters to satisfy the unloaded edge conditions of zero moment and zero reaction shearing forces. By assuming continuous contact of the plate with the elastic layer, and thus equating both expressions for the common deflection, a linear infinite system in infinitely many unknowns is obtained, the coefficients being complicated infinite integrals involving Bessel functions. One case is completed for the axially symmetric case of a concentrated plate load and a semi-infinite layer $t = \infty$. A brief summary is added of the results obtained for a rectangular plate on a similar subgrade when finite difference methods are employed. D. L. Holl, USA

3035. Fralich, R. W., Mayers, J., and Reissner, E., Behavior in pure bending of a long monocoque beam of circular-arc cross section, *NACA TN* 2875, 33 pp., Jan. 1953.

A simple approximate method is developed for treating the

nonlinear "Brazier problem" of beams having a shallow doubly symmetric circular-arc cross section. The method may be readily extended to other shallow cross sections. Numerical results are presented for relationship between bending moment and curvature and for distortion of cross section and maximum axial stresses. An estimate is made of range where local instability of compressed cover will be critical rather than flattening instability.
F. J. Plantema, Holland

3036. **Watts, G. W., and Lang, H. A., The stresses in a pressure vessel with a hemispherical head,** *Trans. ASME* **75,** 1, 83–89, Jan. 1953.

Paper describes computational procedure to obtain stresses in pressure vessels with hemispherical head. It gives tabulated results of shear, circumferential, and axial stresses divided by the circumferential stress in a thin unrestrained cylinder under uniform pressure. These stress ratios are listed for the junction both in the cylindrical shell and the hemispherical end, for various (diameter/thickness) and (head thickness/shell thickness) ratios. The sense, location, and magnitude of the maximum stresses in the shell are also tabulated.

The results indicate that the maximum stress for the ratios investigated is always in the shell, and its numerical value is only approximately 3% greater than the corresponding circumferential stress in a thin unrestrained cylinder under uniform pressure.
N. Sag, Australia

3037. **Haringx, J. A., Stresses in corrugated diaphragms,** Anniv. Vol. appl. Mech., dedicated to C. B. Biezeno; Haarlem, Antwerpen, Djakarta, N. V. De Technische Uitgeverij H. Stam, 1953, 199–213. Hfl. 20.

Paper is an extension of author's previous article [AMR **4,** Rev. 2861] devoted only to displacements and giving the underlying principles of the calculation. Now he derives formulas for the general stress distribution and the maximum stress, valid for a diaphragm without central rigid plates, clamped at the outer radius, and loaded by uniform pressure. Results can be applied when the thickness of the diaphragm is greater than about 0.002 times the radius and when the wave length of the corrugations is small with respect to the radius. However, the latter requirement is not stringent as it turns out in a special case, where maximum discrepancies between author's approximations and exact results amount to 10%.
M. Kuipers, Holland

Buckling Problems

(See also Revs. 3009, 3035)

3038. **Ketter, R. L., Beedle, L. S., and Johnston, B. G., Column strength under combined bending and thrust,** *Weld. J.* 31, 12, 607s–622s, Dec. 1952.

Wide-flange columns made of low-carbon structural steel were tested with applied end moments. Two loading conditions with single curvature, and two with double curvature were tested. Theoretical formulas were derived for interaction curves for initial yield and ultimate collapse.

Comparison with design code of American Institute of Steel Construction shows varying safety factors for the various end conditions. Distribution of bending moment along column had appreciable effect upon strength and type of failure. Some failures occurred due to lateral-torsional buckling, while others were due to local buckling of flanges. Columns in which the theoretical collapse load, based on simple plastic theory, was attained showed failure by local instability. Columns tested at high axial load did not reach theoretical strengths due to residual stress. The theoretical yield strength was attained in all but one test.
S. U. Benscoter, USA

3039. **Salerno, V. L., Bauer, Frances, and Sheng, J., The behavior of a simply supported column under constant or varying end load with transverse displacement of one point of support,** Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 425–434, 1952.

The time variation of lateral deflection for a slender column loaded beyond the Euler load is investigated for three types of loading; $\sigma = \text{const}$, $\sigma = ct$ (c is a constant and t is time), and in the third case, the loading is that resulting from a uniform rate of descent of the loading head of a testing machine. The mathematical solutions are given for all three cases. (Buckling is assumed to be initiated by a sudden displacement of one end of the perfectly straight column.) The numerical examples worked out for the first two types of loading are of considerable interest.
Wm. J. Carter, USA

3040. **Flint, A. R., The stability and strength of stocky beams,** *J. Mech. Phys. Solids* **1,** 2, 90–102, Jan. 1953.

Author presents experimental data for sidewise buckling of I-beam bent in its plane of maximum stiffness by pure end couples. Comparison of test results is made with theoretical critical bending couple of Timoshenko, modified for case of partial plastic condition of beam. Tests were made on small I-beams of mild steel and of aluminum alloy. Steel represents a material of discontinuous slope of stress-strain curve, while the aluminum represents one of continuous slope. The critical couples were all less than those which would produce fully plastic state. However, buckling of the shorter beams occurred with fully plastic condition in the flanges. Theory predicts a lower limit to the range of possible collapse loads given by experiment. Instability concept of Shanley is considered in the paper.
W. H. Hoppman, II, USA

3041. **Seide, P., The effect of lonigitudinal stiffeners located on one side of a plate on the compressive buckling stress of the plate-stiffener combination,** *NACA TN* 2873, 66 pp., Jan. 1953.

Author determines theoretically (minimum potential energy) the effective moment of inertia of the stiffeners to be used in the buckling charts of *NACA TN* 1825 [AMR **3,** Rev. 456]. Some assumptions used are: Zero effect from rivet spacing and rivet flexibility; zero stiffener flexural stiffness for bending parallel to plane of plate; zero stiffener torsional stiffness; zero effect of local buckling of stiffener; small deflection theory for plate, elementary beam theory for stiffeners, elastic range (constant E) with same E for plate and stiffener; and a special boundary condition on transverse edges of the plates (free before buckling, restrained after buckling).
B. E. Gatewood, USA

3042. **Plantema, F. J., and van Alphen, W. J., Compressive buckling of sandwich plates having various edge conditions,** Anniv. Vol. appl. Mech., dedicated to C. B. Biezeno; Haarlem, Antwerpen, Djakarta, N. V. De Technische Uitgeverij H. Stam, 1953, 133–148. Hfl. 20.

Authors use method of partial deflections (separating shear and bending displacements) to find buckling stresses of long sandwich plates with long edges simply supported or clamped. They generalize AMR **5,** Rev. 3069 by removing all restrictions on the partial deflections, thus allowing three boundary conditions per edge. Results show that the simplified analysis is completely adequate for clamped plates but gives errors up to 8% for simply supported edges.
K. H. Griffin, England

3043. Pochop, F., On the stability of long equidistantly transversely stiffened rectangular plates (in German), *Öst. Ing.-Arch.* **6**, 5, 387–402, 1952.

By the energy method, solutions are obtained for two possible loadings: linear variable normal stresses on the ends, and shear. Transverse stiffeners are considered unloaded prior to buckling, and their torsional stiffness is neglected. In two examples, the "minimum rigidity" of the stiffeners is determined. Author shows that good convergence of the approximate solutions can also be obtained for long plates. M. Botman, Holland

3044. Eringen, A. C., Bending and buckling of rectangular sandwich plates, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 381–390, 1952.

Approximate theory is developed from minimum energy principle for behavior of flat sandwich plate under combined lateral load and edge thrusts. Core and faces are assumed homogeneous. Bending as well as membrane stresses are considered for faces; and for core, both bending and shear are taken into account approximately by assuming core displacements to vary linearly with thickness co-ordinate.

Differential equations and boundary conditions are deduced for core and face deflections. Special case is solved numerically, that is, a plate with all four edges of each face simply supported and loaded laterally by an arbitrarily distributed load on upper face, and subject to edge compressions on both faces in only one direction. Critical stresses are computed and presented graphically for (a) bucking of plate as a whole, and (b) buckling by approach of one face toward the other, flattening of core being considered. Buckling of type (a) occurs for ratios of face-to-core thickness higher than a certain limiting value, while that of type (b) occurs for ratios lower than this. Comparisons with Reissner's finite deflection theory are made for type (a) buckling [AMR **1**, Rev. 1108]. Agreement is quite good. No experimental results are given. H. J. Plass, USA

3045. Popov, S. M., Stability of simply supported plates beyond the limit of elasticity (in Russian), *Inzhener. Sbornik, Akad. Nauk SSSR* **9**, 65–98, 1951.

Il'yushin's theory of elastic-plastic stability [AMR **4**, Rev. 2913] is applied to a rectangular plate compressed in one and in two directions. Results are in good agreement with experiments made by author. J. M. Klitchieff, Yugoslavia

3046. Langhaar, H. L., On torsional-flexural buckling of columns, *J. Franklin Inst.* **255**, 2, 101–112, Feb. 1953.

Basing his analysis on work of Dr. Cornelius Lanczos pertaining to problem of buckling of cylindrical shells under axial compression [Purdue Univ., 1943], author devises test for buckling criteria, using the second variation of the potential energy of a mechanical system.

A stationary conservative mechanical system is in stable equilibrium if its potential energy V has a relative minimum, i.e., $\Delta V > 0$ for any small virtual displacements of the system consistent with the restraints. For an elastic system, $V = U + W$, where W is the potential energy of the external forces, and U is the strain energy. Then $\Delta V = \Delta U + \Delta W$. For a system with linear elasticity, the total variation of U is

$$\Delta U = \delta U + (1/2!)\delta^2 U + (1/3!)\delta^3 U + \ldots$$

The potential energy W is taken as a linear function of the virtual displacements so that $\Delta W = \delta W$, then

$$\Delta V = \delta W + \delta U + (1/2)\delta^2 U + \ldots$$

For equilibrium configuration, virtual work principle gives $\delta U + \delta W = 0$. Then for small virtual displacements, $\Delta V = 1/2\,\delta^2 U$ and, since for equilibrium $\Delta V > 0$, it follows that a necessary and sufficient condition for stability is that $\delta^2 U > 0$ (positive definite) for all virtual displacements consistent with the restraints.

The external loads on an elastic system are determined completely by their magnitudes. For small values of the loads, $\delta^2 U$ is positive definite, but for large values $\delta^2 U$ is indefinite. As an illustration, the buckling load P_{cr} is the value of P for which $\delta^2 U$ becomes indefinite. The positive definite criterion of the second variation of the strain energy is used by the author in his analysis.

Application is made to the pin-ended column, clamped-end column, and to pure torsional buckling, and comparison made with Kappus' theory and Wagner's formula.

The definition and use of the second variation may be found in the book by Lanczos, "The variational principles of mechanics," University of Toronto Press. S. Sergev, USA

Joints and Joining Methods

(See also Revs. 3073, 3080, 3081)

3047. Graf, O., Fatigue tests in bending of welded beam to girder connections (in German), Berlin, Springer-Verlag, 1952, Bericht. Deut. Aussch. Stahlbau H. 17, 1–19. DM 10.

3048. Munzinger, F., Experiments with Ellira welding (in German), ibid., 20–48.

Welded connections of standard rolled beam and girder connections were tested under bending loads, cycled 210 times per min. The first six specimens had no tie plates and no seats. The conclusion from these tests was that the tension flange should be connected by butt weld, and fillets are satisfactory in the connection of the compression flange and of the web.

Other specimens with tie plates (passing through slot cut in the web of the girder in some cases) and seats showed that, in all cases, the various notch effects of cuts, corners, and transitions were the determining factor.

The second paper reports on tests carried out from 1939 to 1944 which compared a certain Ellira welding process with standard are welding. D. Vasarhelyi, USA

3049. Privoznik, L. J., Effect of stresses on strength of circumferentially welded cylinders, *Welding J.* **31**, 12, 587*s*–595*s*, Dec. 1952.

Author studies determination of effect of residual stresses upon mechanical properties of circumferentially butt-welded mild steel cylinders tested under high internal pressure at warm temperature (25 C) and at −40 C. The behavior of the as-welded and the stress-relieved vessels is practically the same if there is no longitudinal restraint. Residual stresses are relieved by first pressure cycle yielding, occurring at pressure below that corresponding to yield strength of material. Removal of weld reinforcement decreases strength of both as-welded and stress-relieved vessel.

Schematic diagram of interesting equipment is given. Diameter of cylinders was 6.60 in. and 10.03 in.

W. Soete, Belgium

3050. Bailey, E. C., Schroeder, H. C., and Carlson, I. H., Operating experiences with mechanical joints in high-pressure high-temperature steam piping, *Trans. ASME* **75**, 1, 97–101, Jan. 1953.

Because of the uncertain life expectancy of austenitic-ferritic steel welds for joining the 2¼% chromium, 1% molybdenum alloy-steel main steam piping to Type 347 stainless alloy steel at

the throttle valves of the turbines, two types of mechanical transition joints were installed in the Ridgeland Station of the Commonwealth Edison, Chicago, Ill. The joints, the pressure-seal, and the bellows-type were designed for operation on steam at 1800 psig and 1050 F. These joints have operated satisfactorily since the units were put in service. The pressure-seal joints have been in service a total of 1 year and 5 months, and the bellows-type joints, 1 year. From authors' summary

Structures

(See also Revs. 3008, 3019, 3024, 3026, 3027, 3032, 3036, 3038, 3039)

3051. **Siess, C. P., and Veletsos, A. S., Distribution of loads to girders in slab-and-girder bridges: Theoretical analyses and their relation to field tests,** *Nat. Res. Counc. Highway Res. Bd. Res. Rep.* 14-B, 58–74, 1952.

Simplified treatment of a complex structure of many variables, based largely upon "Moments in I-beam bridges" by N. M. Newmark and C. P. Siess [*Univ. Ill. Bull.* 336, 1942]. One valuable feature of this generally excellent paper is the orderly listing of the multitude of variables involved. Theoretical effects of relative longitudinal-to-transverse stiffness, girder spacing-to-span, type and transverse location of load, stiffness and location of diaphragms are discussed. Use of analyses in planning and interpreting field load tests of actual bridges and theoretical limitations imposed by neglected variables are also discussed. While scope of paper purposely does not include some important items such as deflections and slab moments, paper nevertheless is a concentrated capsule of information on general structural action that could be read by most practicing bridge engineers with profit.
G. P. Fisher, USA

3052. **Viest, I. M., Siess, C. P., Appleton, J. H., and Newmark, N. M., Studies of slab and beam highway bridges, Part IV. Full-scale tests of channel shear connectors and composite T-beams,** *Univ. Ill. Engng. Exp. Sta. Bull. Ser.* no. 405, 155 pp., Dec. 1952. $1.

A presentation of experimental data showing the effectiveness of welded channels as shear connectors in effecting composite beam construction. This bulletin extends the work previously reported in *Bull.* 396 [AMR **5**, Rev. 2331]. Tests on individual connectors show that they act like dowels and will develop bearing pressures several times the concrete cylinder strengths. The load capacity of the connector is proportional to its width, increases nonlinearly with the thickness of the channel flange and web, and is not affected by its height.

Test data are given for four full-scale composite T-beams, designed for highway loadings, showing almost complete interaction between the steel beam and the concrete slab up to the yield point of the steel beam, and an ultimate load capacity close to that computed for complete interaction. Qualitative agreement is demonstrated between the individual shear-connector test and the full-scale beam tests. I. A. Benjamin, USA

3053. **Fisher, G. P., and Boyer, W. C., Reactions of a two-span, skewed, rigid-frame bridge,** *Nat. Res. Counc. Highway Res. Bd. Res. Rep.* 14-B, 75–85, 1952.

Using experimental method based on the Muller-Breslau principle, the authors have determined forces on (1:60 scale) phenolic laminate models of the subject bridge type. Measured influence values have been compared with values computed in accordance with currently accepted theory (Rathbun's analysis with simplifications by Hodges and Barron). Skew has been varied from 0° to 50°, while square span, width, height, and thicknesses were all held constant.

For vertical and horizontal reaction components, agreement between experiment and theory was found to be very close. Torsional moments of reaction (i.e., about an axis parallel to the square span) showed serious discrepancy, with theoretical values apparently on the unsafe side. Authors suggest that error in theory results from the misapplication of the Saint Venant torsion factor to skewed plates. M. J. Holley, Jr., USA

3054. **Paxon, G. S., Load distribution on highway bridges having adequate transverse diaphragms,** *Nat. Res. Counc. Highway Res. Bd. Res. Rep.* 14-B, 46–57, 1952.

Author concludes as a result of field measurements of girder deflections and stresses in reinforcement that exterior beams may carry as much or more load than do interior ones. He suggests that current AASHO Specification in this regard be revised so that, with whole roadway loaded, each girder will be designed for an equal share of such loading. While reviewer welcomes all simplifications of specifications short of significant inaccuracy, he doubts that test of one 48-ft span, supported by four similar girders joined by a single diaphragm at center line, furnishes quite sufficient evidence for the suggested change.
A. H. Finlay, Canada

3055. **Stephenson, H. K., and Cloninger, K., Jr., Method of converting heavy motor vehicle loads into equivalent design loads on the basis of maximum bending moments,** *Texas Engng. Exp. Sta. Bull.* 127, 568 pp., Oct. 1952. $3.

The book provides long-needed data for the evaluation of heavy truck loads in terms of standard vehicle loadings which are used in the design of highway pavements and structures. The mathematical approach, based on maximum bending moments and on the frequency of application of loads, gives correct equivalent loadings for judging the effect of heavy truck loads on structures. Numerous tables for all types of vehicles producing heavy loadings are provided. Indispensable for the highway departments, the material presented accomplishes a long stride toward a satisfactory solution of the controversial question pertaining to so-called "safe loads." M. V. Smirnoff, USA

3056. **Wiingaard-Andreassen, M., On collar beam roof trusses—theory and experiments,** *Bygnstat. Medd.* **22**, 3, 48 pp., 1951.

Paper deals with the load-carrying mechanism of wooden roof frames and the efficiency of various methods of joining the frame members. The roof bents considered all are of the collar-beam type, consisting of two rafter beams, connected to each other at the supports by a horizontal member, and tied together approximately halfway up the rafter beams by a collar beam. The frame is thus statically indeterminate to the first degree, and the bar stresses can be determined in the usual fashion, assuming (a) Hooke's law, and (b) that the deformations are negligible. However, it is shown that these two assumptions are not permissible. Taking the deformations into account and using the theory of limit design, modified expressions for the bar stresses are derived and their range of validity discussed.

Experiments on full-sized and scale models of roof trusses are described and test results given. In general, the modified formulas give considerably better agreement with measured values than do the usual expressions. In most cases, the modified formulas are practical only for trusses with one redundant member. Finally, author suggests various improvements of the joints of wooden roof frames, and gives some considerations on the factor of safety for this type of design. T. A. Mortensen, USA

3057. Leonhardt, F., Modern trends in steel construction, with specific reference to industrial construction (in German), *ZVDI* **95**, 2, 39–42, Jan. 1953.

3058. O'Sullivan, T. P., Strengthening of steel structures under load, *Proc. Instn. civ. Engrs.* **2,** part I, 1, 76–95, Jan. 1953.

Paper describes a means of strengthening a structure to carry loads greater than those originally designed for. Material was added while the structure was under load, the area of the additional material being calculated on the assumption of no initial stress in the parent member. Subsequent tests checked the validity of the assumption that members strengthened while at working loads exhibit the same yield and failure strength as those strengthened while unloaded. T. Andreopoulos, USA

3059. Hall, A. H., The deformation analysis of tapered swept wings, *Nat. aero. Establ. Canada* LR-44, 15 pp., 12 figs., Dec. 1952.

A method is outlined for applying deformation relations, previously derived by the author for aircraft wings without taper, to the analysis of wings with taper. Wing models of cellulose acetate, swept 0, 30, 45, and 60°, were used to obtain experimental results.

The results indicate that, within the range of taper ratios considered, the over-all effect of root deformation can be closely approximated by considering the wings cantilevered from a "root rectangle," which is determined by a simple geometric construction.

The experimental results show close agreement with theoretical predictions, both for bending and torsional deflections.

Utilization of this method will simplify analysis of swept wings, and the good correlation of test with theory indicates that it may be used with confidence. G. L. Bryan, USA

3060. van der Neut, A., The local instability of compression members built up from flat plates, Anniv. Vol. appl. Mech., dedicated to C. B. Biezeno; Haarlem, Antwerpen, Djakarta, N. V. De Technische Uitgeverij H. Stam, 1953, pp. 175–197. Hfl. 20.

Semi-monocoque structures, used in aircraft and in some types of ships, consist of sheets reinforced by longitudinal stiffeners supported by frames. When loaded in longitudinal compression, structure fails by instability which is classified by author as (a) column failure or Euler instability, (b) torsional instability, (c) general instability due to weakness of frames in bending, and (d) local instability. The coherence of individual plates requires that buckling wave patterns of different individual plates are compatible. Problem of buckling load computation therefore rests on determination of the correct application of plate theory such that interaction of collaborating plates is accounted for adequately. Ordinary engineering methods for finding local buckling stresses of stiffeners comprise in considering each individual plate to have a hinged support at line of intersection of adjacent walls. The buckling stress is then assumed to be equal to a weighted average of unequal buckling stresses of composing plates. Author maintains that this approach is faulty in many respects and can only be used for a preliminary approximation. In fact, reliability of this method decreases further in case of stiffened sheet, where breadth/thickness ratio of panels is much larger than that of stiffener walls. Such instances arise in modern aircraft wings, where compressive load per unit width is very high due to large size of aircraft or small thickness of wings.

Although methods have been evolved for exact determination of stability criterion, they are very lengthy and require considerable laborious computation. The object of this paper is to present a simpler straightforward method of analysis of complicated systems by use of a set of nomograms. Author considers only flat plates stiffened by Z, channel, and square top-hat sections, and excluded rounded top-hat stiffeners or stiffeners with flange edges having bulbs or lips. After setting forth certain assumptions, analysis is given of buckling of a web with edge restraint; buckling of an arbitrary sequence of plates; and extensions to closed and bifurcating systems. Application of method, with numerical example, is also furnished. Method appears to be comparatively more accurate and less laborious than those used up to now for such buckling analysis.

S. K. Ghaswala, India

Rheology (Plastic, Viscoplastic Flow)

3061. Reuss, E., The stress-strain equations of highly viscous fluids and their application to ultrasonics (in German), *Acta Tech. Hung., Budapest* **6,** 1/2, 65–77, 1953.

Author demonstrates that Stokes' stress-strain equations for viscous fluids fail for high viscosities, and suggests their replacement by the Maxwell-Natanson equations, involving a steady transition from the liquid to the solid phase, as occurs with glass, asphalt, etc.

On the basis of these equations, the dispersion and the damping per wave length are calculated for longitudinal and transverse waves. Agreement with experimental results is not satisfactory, but the analysis forms a rigorous basis for a discussion of the discrepancies.

From author's summary by F. R. N. Nabarro, England

3062. Torre, C., Hydrodynamic theory of solids (in German), *Öst. Ing.-Arch.* **6,** 5, 417–428, 1952.

The firmo-viscous Jeffreys solid composes a total tension from an elastic and a viscous part. In this paper, only the influence of the second part is examined, i.e., the influence of the deformation speed on the stresses to which a solid body is subjected. This hydrodynamical theory of solid substances is applied to the investigation of the stresses operating on subterraneous (underground) structures. From author's summary

3063. Engelhardt, W., The cause of the natural yield point (in German), *Technik* **7,** 9, 11; 515–517, 659–662; Sept., Nov. 1952.

Paper contains words, not equations; not experimental or mathematical results, but rather qualitative hypotheses. Past observations and theories are reviewed. Paper emphasizes effects of boundaries between lattice regions (crystals or crystallites) rather than interiors. At boundaries between adjacent lattice regions, mismatching creates anomalous locations lacking clear arrangement. Strong anomalies permit slip on lattice glide planes at low external stresses, weaker anomalies at higher stresses. Gaussian distribution of numbers of anomalies of each strength are assumed for pure metals, with most at low strengths. Natural yield point could occur if this distribution were changed by reducing all anomaly strengths above some critical value down to that value, so slip would not be permitted at stresses below stress corresponding to this critical anomaly strength. Under certain restrictions in supersaturated interstitial alloys, small equilibrium groups of the solute atoms could precipitate, and would preferentially locate around stronger anomaly locations in boundaries. This would "fill the gaps" in the anomaly locations, and would have the required effect of reducing the strengths of anomalies above some critical strength down toward that value. Consequences of this model are discussed.

Reviewer feels paper provides a useful approach to this and related problems, but, as is common to many recent papers about deformations of solids, so many different models are possible that even phenomena which do not occur could be "explained" in the consistent manner. A quantitative theory is sorely needed.

R. B. Green, USA

3064. **Budiansky, B., Dow, N. F., Peters, R. W., and Shepherd, R. P., Experimental studies of polyaxial stress-strain laws of plasticity,** Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 503–512, 1952.

Thin-walled tubes of 14S-T4 aluminum alloy were stressed into the plastic region in axial compression and then twisted. The torque was kept proportional to the change in compressive load over that prevailing when twisting began. In two tests, the compressive load was diminished as twisting proceeded (unloading), and in four tests the compressive load continued to increase (loading). In the unloading cases, there was no additional plastic flow until a certain amount of shear stress had developed, but plastic flow did begin considerably earlier than predicted by the flow theory. (Reviewer's note: this is to be expected for conditions of unloading, because of the Bauschinger effect.)

For conditions of loading, the axial strain conformed fairly well to either the flow theory or deformation theory (their predictions of axial strain differ but little). The shear strain for loading conditions had values lying approximately midway between the predictions of the flow theory and deformation theory, but not very close to either.

J. D. Lubahn, USA

3065. **Hill, R., Lee, E. H., and Tupper, S. J., Plastic flow in a closed-end tube with internal pressure,** Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 561–567, 1952.

Paper first appeared in 1946 as a report from the Theoretical Research Branch, Armaments Research Dept., Ministry of Supply, United Kingdom. In a previous paper, a geometrical method of representing stress and strain in a plane diagram was used to determine the stress and strain distributions in a thick tube subjected to internal pressure when longitudinal strain was prevented. Elastic strains were taken into account in the plastic region. In the present paper, using the same methods, the problem is considered for a closed-end tube with internal pressure. As the flow develops, the longitudinal strain increases toward a limit and the longitudinal stress approaches the mean of the other principal stresses. The error in the longitudinal stress, according to the usual treatment which assumes incompressible material and neglects elastic strains, has a maximum value of 18%, compared with over 60% for the plane strain problem. Use of a deformation-type approximate elastic-plastic law gives an error of only 5% in stresses, but predicts an incorrect longitudinal strain variation.

From authors' summary by W. M. Shepherd, England

3066. **Sidebottom, O. M., and Chang, C.-T., Influence of the Bauschinger effect on inelastic bending of beams,** Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 631–639, 1952.

For shake-down analysis in connection with plastic or limit design of steel structures, an idealized elasto-plastic stress-strain diagram without or with strain-hardening effect, but without Bauschinger effect, is generally assumed. This provides in reverse loading a linear range of stress equal to $2s_e$ (s_e is proportional limit of idealized diagram). Authors report two beam tests, one on SAE 1020 and one on rail steel, both annealed, whose diagram for moderate strains they idealize by an elastic line and a strain-hardening straight line. Moment vs. outer-fiber-strain curves are computed on this basis and compared with test results, obtained in reverse bending, i.e., equal and opposite loading.

Experimental M-e curves for reverse loading and reloading fall below computed curve which authors explain by Bauschinger effect, which results in residual stresses after first loading about 40% smaller than computed. These residual stresses were obtained not by direct measurement but by indirect means. The linear range in reverse bending was found very much smaller than $2s_e$. Hence, these two tests seem to invalidate, at least to some extent, the usual assumptions of shake-down analysis. However, neither of these steels is characteristic of the usual structural steel inasmuch as both are gradual-yielding without sharp break and upper yield point. Also, total amount of maximum fiber strain was limited to $\pm 2e_e$. In view of this, the statement that "the load-carrying capacity of the beams was appreciably lower than the theoretical value," given in the abstract but not elaborated in the body of the paper, does not seem to be supported by the evidence, unless the statement is meant to refer to shake-down strength rather than "carrying-capacity" in the usual sense.

G. Winter, USA

3067. **Barrett, A. J., The bending of some common beam sections into the plastic range,** *J. roy. aero. Soc.* **57,** 506, 110–115, Feb. 1953.

Paper analyzes pure bending of beams about axis of symmetry beyond elastic range. Calculations of bending moment and form factors for various shapes are made, using a mathematical expression for the stress-strain relationship in the form: $e = af + bf^n$, where e is strain, f is stress, and a, b, n are constants (determined from stress-strain curve).

H. I. Fusfeld, USA

3068. **Körner, H., Time-dependent materia stresses. Example: concrete** (in German), *Bauingenieur* **28,** 1, 12–17, Jan. 1953.

A few well-known remarks on Maxwell's relaxation formula.

C. B. Biezeno, Holland

3069. **Marin, J., and Hu, L. W., Deflection of members subjected to bending accompanied by creep,** Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 613–618, 1952.

This article is very interesting in that it shows how tension and compression creep curves can be integrated and applied to the problem of creep in bending. It would be interesting to see a comparison with experimentally measured creep in bending as published in the *Journal of Applied Mechanics*, June 1936.

R. G. Sturm, USA

3070. **Marin, J., and Pao, Y.-H., A theory for combined creep strain-stress relations for materials with different properties in tension and compression,** Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 585–593, 1952.

The essential parts of the paper utilize the theory of states in the plastic or creep range under combined stress of isotropic materials, which is not new and has been published earlier by other investigators of these subjects, to whose work no references can be found in this paper. The authors attempt to discuss a case of creep by assuming that the so-called long-time creep curves taken under a tensile and under a compression stress held constant during a test would differ from each other—a fact that is made very improbable through various carefully interpreted considerations, or through experimental evidences pertaining to isotropic materials which are otherwise also postulated in the authors' theory.

The example of Plexiglas to which they refer does not seem to warrant this entirely avoidable, unnecessary complication with which they have burdened their equations, which otherwise follow channels that have been dug out previously by others whose work was ignored. A. Nadai, USA

3071. Johnson, A. E., and Frost, N. E., The temperature dependence of transient and secondary creep of an aluminum alloy to British standard 2L42 at temperatures between 20° and 250° C and at constant stress, *J. Inst. Metals* **81,** 93–107, 1952/1953.

The effects of temperature on the various phases (transient and steady-state creep) of forward creep and on creep recovery have been investigated for an aluminum alloy to British Standard 2L42 (R.R.59), by means of creep tests in pure torsion on thin-walled tubular specimens at a constant stress of 2 tons/in.^{2}, and at 50 C intervals over the range 20–250 C. The tests were of moderately long duration.

It is concluded that, for this alloy, no current fundamental theory adequately describes the transient creep between 20 and 250 C, although up to 200 C the secondary creep rates agree quite well with theories based on the Eyring process rate conception. Superposition theories do not represent the recoverable creep measured. The total forward creep is well represented at all temperatures by an equation

$$\gamma = K_1 t^{1/m_1} + K_2 t^{1/m_2}$$

where K_1 and $1/m_1$ refer to the nonrecoverable portion of the forward creep strain, and K_2 and $1/m_2$ refer to the recoverable portion of the creep. K_1, K_2, and $1/m_1$ vary with temperature; $1/m_2$ is effectively constant.

From authors' summary by B. G. Rightmire, USA

Failure, Mechanics of Solid State

(See also Rev. 3090)

3072. Woodfine, B. C., Temper—Brittleness: a critical review of the literature, *J. Iron Steel Inst.* **173,** part 3, 229–240, Mar. 1953.

3073. Taylor, B., The strength of large bolts subjected to cyclic loading, *Trans. Inst. mar. Engrs.* **64,** 12, 233–243, Dec. 1952.

Paper appears to be written for a reader interest that has little prior technical background or knowledge regarding the fatigue of materials and mechanical equipment. The factors affecting fatigue life of bolted assemblies are reviewed, including the material, stress concentrations, and practical fabrication and design considerations. A limited number of tests are reported on 3-in. and 1-in. bolts using methods of manufacture and design similar to those commonly used in heavy marine engines of British manufacture. A wide variation in fatigue strength of large bolts having lathe cut threads is reported, with failures at loads as low as 16% of those causing static failure. The tests were too limited in extent to form the basis for very definite conclusions as to the effect of various variables that are discussed by the author. Details of fatigue testing equipment as well as photographs of fatigue fractures in large bolts are included. B. G. Johnston, USA

3074. Yokobori, T., Failure and fracture of metals from the standpoint of the stochastic theory, *J. phys. Soc. Japan* **8,** 1, 104–106, Jan.-Feb. 1953.

Paper consists of brief discussions of various failure phenomena (fatigue, fracture, creep, ductility transition, fluctuations in strength) from the standpoint of probability theory, reference being made mainly to previous publications by author.

A. M. Wahl, USA

3075. Yokobori, T., Ductility transition and cold brittleness, *J. phys. Soc. Japan* **8,** 1, 107–109, Jan.-Feb. 1953.

The occurrence of the change of fracture from ductile type to a brittle one which is often called "cold brittleness" is studied from the standpoint of the stochastic theory. In the present paper, failure and fracture processes were treated as reaction-rate process. The relation between deformation velocity V_B and the absolute temperature for brittle fracture was obtained as the form: $\log V_B = -(Q/RT) + c_0$. The effect of radius of notch on the transition, the size effect, and the scatter of the brittle strength were also interpreted from the same standpoint. In the case in which the Cottrell-Bilby mechanism for yielding, and the nucleation process for brittle fracture, respectively, are regarded as rate-determining, similar results can also be obtained.

From author's summary

3076. van der Veen, J. H., Development of a testing-method on brittle fracture of mild steel plates, Symp., Notch bar testing and its relation to welded constr., Lond., Inst. Weld., 35–43, 1953.

A slow notched-bar bend test is described. The specimen is a strip 70 mm wide cut from steel plate of full thickness, provided with a 3-mm deep 45° V-notch running perpendicular to the plate on one side of the strip. This is supported on the notched side symmetrically by two cylinders 200 mm apart and bent in its own plane by pressure applied opposite the notch. Suggested criteria for the tendency of the material to brittle fracture are: (1) The maximum load in the test and the deflection and energy absorption at maximum load; (2) the length of the fibrous crack that introduces cleavage; (3) the energy absorbed after beginning of the cleavage crack (substantially, the work absorbed by the shear lips if such are present).

The results do not differ substantially from those of other tests of similar type. E. Orowan, USA

3077. van Wijk, F., The influence of strain in the artificial aging of steel (in Dutch), *Metalen* **8,** 1, 1–6, Jan. 15, 1953.

The decrease of Charpy-keyhole impact value by artificial strain-aging was determined for a $^1/_2$-in. mild steel plate of high weldability. Independent variables were the tensile prestrain (0, 1, 2, 3, 5, and 10% respectively) and the test temperature (−40, −20, 0, +20 C). Between prestraining and impact testing all specimens were held for half an hour at 250 C. The impact value after aging was found to decrease with increasing prestrain, and with decreasing test temperature. Curves of impact value vs. percentage strain for each of the applied test temperatures show that from 0 up to about 3% strain the effect of aging is proportional with the strain. Beyond this point, slopes of the curves were found to decrease. A statistical consideration is given which shows that one must be very careful when rejecting material on the ground that the percentage decrease in impact value after aging is above a specified limit, because the accuracy of this quantity, when determined by a rather small number of specimens, is relatively low. When only three tests are made before and after aging, the decrease in impact value must exceed 32.3% in order to conclude with a statistical significance of 95% that the average decrease of the plate tested will be more than 25%. J. H. van der Veen, Holland

3078. Roberts, C. W., Intercrystalline corrosion in cast zinc-aluminum alloys, *J. Inst. Metals* **81,** 301–309, 1952/1953.

To determine the susceptibility of zinc-aluminum alloys to intercrystalline corrosion in an air/water-vapor atmosphere at 95 C, and to examine the effect of the presence of other elements on the incidence of this form of corrosion, alloys of various compositions within the range aluminum 0–22%, copper 0–1.5%, magnesium 0–0.09%, lead, tin, and cadmium 0–0.030%, bismuth 0–0.016%, and manganese 0–0.050% were prepared and tested in the as-cast condition.

The main conclusions drawn from the work are: (1) Intercrystalline corrosion is confined to the α (zinc-rich) phase, although attack is more severe when the β (aluminum-rich) phase is also present, as a result of the larger surface area of the α grains in the two-phase alloys; (2) the severity of attack is greatly increased by the presence of small percentages of lead, tin, cadmium, and bismuth; (3) the addition of a small amount of magnesium greatly reduces the severity of the corrosion, whether impurities are present or not, provided conditions are such that intermetallic compounds of magnesium with the impurity elements are not formed; and (4) the presence of copper increases the resistance of two-phase alloys to intercrystalline attack.

The results of the present investigation and those published by other investigators are discussed, and tentative theories are put forward to explain certain aspects of the phenomenon.

From author's summary

3079. **Mohaupt, A. A., and Heebink, B. G., Effect of defects on strength of aircraft type sandwich panels,** ASME Ann. Meet., New York, Dec. 1952. Paper 52—A-99, 10 pp., 5 tables, 5 figs.

Poor bonds in sandwich constructions of aluminum facings on balsa core and of glass-cloth facings on balsa core reduced the flatwise tensile strength considerably more than they reduced the bending and edgewise compressive strengths.

Wrinkles in the glass-cloth facings of specimens having balsa or glass-cloth honeycomb cores reduced the bending, edgewise compressive, and longitudinal tensile values by amounts in proportion to the depth of the wrinkles. A fold in the glass-cloth facing caused considerable reductions in bending, edgewise compression, and longitudinal tension. From authors' summary

Material Test Techniques

(See also Revs. 3071, 3076)

3080. **Apblett, W. R., Jr., Felmley, C. R., and Pellini, W. S., Factors which determine the performance of aluminum-alloy weldments,** *Welding J.* **31,** 12, 596s–606s, Dec. 1952.

The "explosion bulge test" has been utilized to establish the factors which determine the performance of aluminum-alloy weldments. Two types of alloys were investigated: (1) The common heat-treatable type alloys (61S and 24S), and (2) the solid-solution hardening type containing Mg as the primary alloying element. The deformation characteristics of the various weld and base-metal combinations were determined by means of photogrids applied to the surface of the specimens. Tensile test data of the weld joints and parent plate were also obtained in order to establish the significance of the deformation characteristics.

It is indicated that the inability of weldments of heat-treatable type alloys to develop appreciable deformation is due to strain concentration in the low-strength weld and heat-affected zone. Weldments of the solid-solution hardening type alloys are capable of developing extensive deformation prior to failure due to a more desirable distribution of strain in the weld, heat-affected zone, and parent plate. This is the result of a closer matching of the strengths and ductilities of the various components of the weldment, which is possible by the use of these alloys.

From authors' summary by H. A. Lepper, Jr., USA

3081. **Werthebach, P., Tests on welding materials and steel with respect to the tensile strength of welding seams** (in German), *Stahl u. Eisen* **73,** 2, 84–91, Jan. 1953.

In the course of the procedure proposed by K. Kautz for the determination of the degree of welding-seam-crack tendency of filler materials used for fusion welding, two cylinders of 50-mm diam and 50-mm length from normalized plain steel with a strength of 50 to 60 kg/mm² are slightly pressed against each other and welded on either side. First, two welding layers are provided on one side, one immediately after the other; then a third layer is provided on the other side. As the shrinking process of the first two layers is not yet ended when the third layer is applied, the latter is exposed to tensions and will crack if the filler material inclines to high-temperature cracking.

The above-mentioned procedure was generally confirmed by austenitic filler materials. The present new examination was intended to show to what extent the material of the cylinder and the other conditions of the experiment are able to influence examination results. Experiments were made with 25 different filler materials and specimens from 7 types of plain steel, the carbon percentage of the latter varying from 0.04 to 0.71. The experiments proved that the examination results were comparable only if the conditions of the experiment were strictly observed. Apart from that, they proved the importance of the cylinder welding test for judging the filler materials. E. Siebel, Germany

3082. **Dickinson, C. J., and Hadley, H. R., Pressure tests on flat aluminium alloy panels,** *J. roy. aero. Soc.* **57,** 505, 46–49, Jan. 1953.

Test results on flat square panels with fixed edges and subjected to normal pressure are compared with theoretical solutions extended from data in Royal Aeronautical Society Stressed Skin Data Sheet 02.09.02. Failure to obtain satisfactory edge fixity required that initial test setup be revised. Subsequent tests with improved edge fixity produced excellent agreement with theoretical deflections, but measured strains at center and edge were 80 to 95% of theoretical values. Reviewer believes that discrepancies of this magnitude may be attributed at least in part to continued difficulty of maintaining edge fixity and in measuring strains adjacent to fixed support. W. E. Black, USA

3083. **New. J. C., A nondestructive differential-pressure test for thin shells,** *J. appl. Mech.* **20,** 1, 48–52, Mar. 1953.

Liquid-filled thin shells are subjected to external pressure under close control of both internal and external pressures. At the point of incipient buckling the difference between the two pressures remains constant despite external pressure increase. At yielding, this pressure differential decreases. In each case, shell deformations are so small that the shell is restored to its original shape on release of pressure. Experimental verification checks conventional pressure tests within 10%.

Method appears particularly applicable for routine and exploratory tests of buckling loads of shells where re-use of the shell is mandatory. Author mentions several other uses and also discusses the limitations and experimental difficulties.

E. G. Chilton, USA

3084. **Ramberg, W., and Miller, J. A., Stress-strain relation in shear from twisting test of annulus,** *J. Res. nat. Bur. Stands.* **50,** 2, 125–130, Feb. 1953.

Presentation of a method for obtaining stress-strain data in shear for an isotropic plate. The paper is introduced by an analytical justification of the method and a demonstration of the limitations placed on the design of the test specimen. The test is prepared essentially as follows: The test specimen, a circular

disk of radius C, is cemented, concentrically, to the end of a circular shaft, radius A, and a rigid circular ring of inner radius B, where $A < B < C$. This assembly, with a suitable jig, is placed in a torsion testing machine. A twisting moment applied to the assembly will cause a shearing deformation in the plane of the plate and hence a relative angular displacement between the solid shaft and the ring.

A gage was designed to measure the relative angular displacement between two circles on, and concentric with the test plate. The data, relative angular displacement vs. applied moment, yield the gradient in displacement with moment. With this information and an analytically derived relationship, a step-by-step construction of the shear stress-strain diagram is possible. This construction must be started, however, with the entire plate within the elastic range. The results obtained on aluminum sheet 75S-T6 are in excellent agreement with those obtained from simple tension and compression tests. The comparisons were made on the basis of the octahedral shear theory. G. H. Lee, USA

3085. Johnson, A. J., and Richards, J. E., The structural behaviour of the main engine seatings and bedplate in a cargo ship under static bending tests, *N. E. Cst. Instn. Engrs. Shipb. Trans.* **69,** part 2, 45–90, Dec. 1952.

Paper gives the results of an investigation into the flexure of the bedplate and seatings of the triple-expansion steam engine of a standard type of 10,000-ton dry-cargo ship while subject to static bending tests. Details are given of the instrumentation and testing technique used in the investigation. Gage readings were recorded for eleven different conditions of loading of the ship.

The results show that the vertical deflection of the bedplate is approximately twice that calculated for the flexure of the ship girder over the same length, and that there is no evidence of sliding movement between the bedplate base and the seating. It is shown that the comparatively large deflection of the bedplate is due to the local changes of section in way of the engine seating. This effect is demonstrated by the mathematical solution of an analogous problem, that of a composite beam subjected to end loading and a uniform lateral elastic restraint.

The vertical deflections of the bedplate under service conditions are estimated from the results of the investigation. Assuming true alignment in the light condition, the central deflection of the bedplate in a typical loaded condition is estimated to be 0.010 in. sag in still water. During the passage of the ship through a standard L/20 wave there will be superimposed a deflection of ± 0.0137 in. so that the actual central deflections will be from 0.037 in. hog to 0.0237 in. sag. Provided that there is no uneven wear-down of the main bearings, the additional bending stress in the crankshaft is small. Deflection of the bedplate causes uneven load distribution and consequent unequal wear-down between the main bearings, however, and it is probable that this is the most serious effect of the flexure, resulting in high crankshaft bending stresses. From authors' summary

3086. Tucker, G. E. G., and Murphy, P. C., A method of determining orientations in aluminum single crystals and polycrystalline aggregates, *J. Inst. Metals* **81,** 235–244, 1952/1953.

A method is described for constructing (100) pole figures from angular measurements of the etch pits that are developed in micrographically prepared commercial and super-purity aluminum. Suitable etching reagents are indicated, and the technique of angular measurement using a metallurgical microscope is discussed in detail.

A table of angular values which reduces the calculation necessary in plotting the stereographic projections from the etch-pit data has been constructed and is reproduced.

It has been found that the pole figures prepared by this method agree well with those produced from data obtained by x-ray-diffraction techniques. From authors' summary

3087. Matthews, J. B., The measurement of the relative hardnesses of fine powder particles, *J. Inst. Metals* **81,** 279–285, 1952/1953.

The experimental technique originally devised by Chalmers [title source, **67,** 295, 1941] for the determination of the surface hardness of metallic strips, by their bombardment with a hard powder such as sand or carborundum, has been applied to the measurement of the relative hardnesses of different powder particles. The method depends on the decrease in specular reflectivity produced by the indentation of the reflecting surface by the bombarding powder particles, when the hardness of the surface is not far removed from that of the particles. Formal relationships have been derived connecting the decrease in reflectivity with the relative "hardness" of the particles.

From author's summary

Mechanical Properties of Specific Materials

(See also Revs. 3047, 3048, 3071, 3072, 3078, 3086, 3087, 3101, 3214)

3088. Keeler, J. H., and Davis, H. M., Density and hydrogen occlusion of some ferrous metals, *J. Metals* **5,** 1, 44–48, Jan. 1953.

The high-purity iron showed neither a density decrease nor any noteworthy change in hydrogen occlusion with cold reduction or annealing. Apparently the presence of impurities is necessary for a density decrease with cold reduction.

Cold reduction decreased the density of ingot iron but did not change the hydrogen occlusion of the metal. Annealing after cold reduction restored only 42% of the original density loss and did not influence the hydrogen occlusion.

Cold reduction decreased the density of SAE 1020 and increased the hydrogen occlusion at 250 C about 100-fold. This remarkable hydrogen occlusion decreased at higher temperatures, but remained greater than that of the unstrained material. Annealing diminished the large occlusive capacity caused by cold work. The hydrogen-occlusion behavior of the SAE 1020 is not compatible with the theory of occlusion by storage of molecular hydrogen.

To reconcile the various observations, it is proposed that two varieties of voids result from imperfect plastic flow of the ferrite about inclusions or other phases during cold rolling.

From authors' summary

3089. Wyatt, H. W., Bolton, J. W., and Steinbuch, M. L., Comparisons at elevated temperatures of some commercial grades of ferritic cast steels, *Trans. ASME* **75,** 2, 289–298, Feb. 1953.

This paper presents some direct elevated-temperature test comparisons of cast steel grades WC4, WC5, WC6, and WC9 of ASTM Specification A217-49T. The materials studied are from commercial heats. Care has been taken to hold all variables in manufacture, treatment, and composition (aside from prescribed "alloy elements") as closely alike as possible. The tests show that at the temperatures under consideration, the nominal 1% molybdenum grades WC5 and WC9 are higher in "creep strength" than the nominal $\frac{1}{2}$% molybdenum grades WC4 and WC6. Grade WC5 has shown, in this direct comparison, higher "stress-rupture strength" than the other three grades.

Authors also include data on the impact and hardenability characteristics and short-time elevated tensile test measurements

of these cast steels. The order of "safest" to weld of these cast steels is WC4 > WC5 and WC6 > WC9. Graphitization was absent in any of the four grades of steel studied, and it was further shown that the microstructure of these steels was stable, greatly enhancing the reliability of the performance data and conclusions given in this paper.

From authors' summary by R. Speiser, USA

3090. **Liu, S. I., and Ripling, E. J., Flow and fracture characteristics of the aluminum alloy 24S-T4 as affected by strain thermal history,** *J. Metals* **5,** 1, 66–68, Jan. 1953.

Paper reports results of tests designed to establish the flow and fracturing characteristics of metals prestrained under one set of thermal conditions and subsequently tested under a second set. Particular conditions studied were low temperatures (−196 C and −78 C) and room temperature. Aluminum alloy 24S-T4 was selected as the material to test, since the transition temperature effect could thereby be avoided in low-temperature conditions. Results appear to warrant extension of previously reported flow and fracture characteristics at elevated temperatures to the low-temperature ranges. [See *Trans. AIME* **185,** p. 78, Feb. 1949; AMR **3,** Revs. 62, 1921.] W. D. Jordan, USA

3091. **Archard, G. D., A method of preventing mercury from sticking to glass,** *J. sci. Instrum.* **29,** 12, p. 412, Dec. 1952.

It is possible to make mercury run freely on glass by first placing it in contact with aluminum for a short time. Samples of aluminum and of clean distilled mercury, obtained from two independent sources, were found to produce this effect, no matter whether the aluminum was in the form of foil or an evaporated layer. An hour's contact with aluminum foil, which was occasionally shaken, sufficed to condition 1 cc of mercury and insure that any small drop separated from it by a sharp blow should roll freely. It has been found that a sample of treated mercury retained this property after storage in a specimen tube for four months. Nevertheless, the property may be destroyed at will merely by squeezing the treated mercury through a linen handkerchief. From author's summary

3092. **Saretok, V., Admixtures of concrete** (in Swedish), *Trans. Chalmers Univ. Technol., Göteborg* no. 127, 67 pp., 1952.

Paper is extensive review of present knowledge of admixtures to concrete. Early chapters summarize chemistry of reactions taking place in hardening concrete. Admixtures are then classified as (1) setting time regulators, (2) surface-active agents (dispersing agents), (3) air-entraining and foaming agents, (4) waterproofing materials, (5) filler-type admixtures (inert fillers and pozzolans), (6) miscellaneous.

A section is devoted to each type in which the action of the different materials mentioned is discussed in its effect on the chemistry of the hardening reactions mentioned earlier.

Paper is very comprehensive in materials studied, and does very good service in summarizing knowledge to date of admixtures to concrete. An English summary and an extensive bibliography including patents are attached. A complete English translation would benefit many. F. A. Blakey, Australia

3093. **Bartenev, G. M., Theory of the deformation of rubber** (in Russian), *Zh. tekh. Fiz.* **22,** 7, 1154–1165, July 1952.

In this work, originating from the scientific institute of research of the rubber industry, a short treatment of the elastic deformation of rubber is followed by a discussion of the high elastic state. In this state a chain denotes a linear section of macromolecules between two nodes of the three-dimensional molecular net. A statistical view of the chains leads to the strain $\lambda = \lambda_\infty \mathfrak{L}(\omega + pf\lambda/kT)$ with $\omega = \mathfrak{L}^{-1}(1/\lambda_\infty)$, $\mathfrak{L}^{-1}$ the inverse function of the Langevin function $\mathfrak{L}$, f the force referring on the initial area, i.e., $\sigma = \lambda f$. It is supposed that each chain is in a uniaxial state of stress, making an angle ϑ with its direction, and that during stretching the state remains amorphous.

A sample with 2% sulphur agrees with experiment in the case of 80 C, both when the strain is measured 2 hr after loading and when the stretched sample remains 2 hr at 80 C; then the temperature changes repeatedly between 60 and 80 C. The mechanical reversible relaxation diminishes to zero after 3 hr, and then only the irreversible relaxation remains, which varies in a linear manner since the reversible stress hardly alters during this time. λ_∞ refers, therefore, to the initial state of the three-dimensional net.

The modulus of elasticity E_0 is dependent on the preceding high elastic state. One measures for $\lambda_r = 8$ (strain of 700%) after freezing in liquid nitrogen a value of 25,000 kg/cm², σ (rupture) = 2000 kg/cm², λ (rupture) = 9.75. With σ (rupture) = $E_0[\lambda(\text{rupture}) - \lambda_\infty]/\lambda_\infty$, one finds $\lambda_\infty = 9.02$. Since at low strain, $\sigma = E_\infty(\lambda - 1)$ with $E_\infty = 3kT/p\lambda_\infty$, E_∞ can be inferred from a $\lambda - \sigma$-diagram; for the said sample one has $E_\infty = 11$ kg/cm² and $p = 1.49 \times 10^{-21}$ cm³. Herefrom, an area s of the chain segment is 9 or 12.10^{-15} cm², i.e., since $p = s.l$, a length l of 12 or 16 Å. Finally, the dependence of the constants λ_∞ and E_∞ of rubber upon the number of chains per ccm is discussed.

H. Schlechtweg, Germany

3094. **Schmieder, A. K., and Dikeman, H. G., Recent developments in plastics of interest to mechanical engineers, based on a survey of the periodic literature from July 1951 through June 1952,** ASME Ann. Meet., New York, Dec. 1952. Paper 52—A-169, 16 pp.

3095. **Pironneau, Y., Deformation of single crystals of Al-Cu by repeated shocks** (in French), *C. R. Acad. Sci. Paris* **236,** 1, 46–48, Jan. 1953.

Author describes experimental apparatus for deforming specimens (in this case, single crystals of Al-4% Cu) by rapid extensions. The equipment is capable of imparting a maximum velocity of 33 m/s to the specimen at the instant of the shock. Author concludes that the elongation of the specimen is proportional to the number of shocks; hence, to the energy absorbed. This note describes results from only a few experiments. However, evidence from x-ray diffraction and micrographic studies is cited to support the observations made on the deformed specimens. A. D. Schwope, USA

Mechanics of Forming and Cutting

(See also Revs. 2990, 3213)

3096. **Krekeler, K., The machining of metals and nonmetals [Zerspanbarkeit der metallischen und nichtmetallischen Werkstoffe],** Berlin, Springer-Verlag, 1951, xii + 358 pp. DM 34.50.

Book is most recent of five books published in Germany since 1949 in the field of metal cutting. It contains 24 chapters covering following topics: Effects of alloying materials on workpiece and tool material properties; cutting tool materials, including carbides and diamonds; normal and quick methods of determining machinability; machining operations, i.e., turning, drilling, boring, reaming, milling, sawing, filing, broaching, grinding, polishing, and superfinish; calculation of cutting speeds and forces; tool angles; use of negative rake angles; effect of physical properties and metallurgy on machinability; machining of

steels, magnetic materials, weldments, forming rolls, copper, zinc alloys, aluminum, magnesium alloys, natural rocks and stones, wood, glass, earthenware, porcelain, and synthetic materials (plastics, etc); and effect of coolants on machinability.

Book is full of valuable data on machining practices established in Germany and is not intended to cover either the theory or the physics of metal cutting. D. Kececioglu, USA

3097. Hahn, R. S., Metal cutting chatter and its elimination, ASME Ann. Meet., New York, Dec. 1952. Paper 52—A-41, 10 pp., 14 figs.

Chatter, the periodic surface irregularity occurring when machining materials, may be forced or self-excited. The first kind is related to the speed of the spindle, the second kind is independent of machine speed. This paper treats only the second kind. Three phenomena are discussed, distinguished as primary chatter, transitional instability, and feedback effect.

It is proved by experiments that primary chatter is not due to a negative force-cutting speed characteristic; connection to lag of the temperature and stress state is believed. Transitional instability is associated with a work-hardened skin originating from the previous passing of the cutter and which has to be broken before another chip springs off. Feedback effect is due to the chatter of the previous revolution of the workpiece, producing a periodically varying force on the tool. It may sometimes be suppressed by making one cutting edge follow another at $(n + 1/4)$ wave lengths.

The periodicity of the undulatory surface is greatly determined by the natural frequencies of the tool-supporting structure. After having found the mode of vibration that is responsible, chatter may be suppressed by applying a mechanical damper to the corresponding structure element.

C. Zwikker, Holland

3098. Herrmann, O., The optimal aperture angle of wire-drawing stones (in German), *Werkstatt u. Betr.* **85,** 7, 291–293, July 1952.

Author draws several consequences of a formula of Siebal for the drawing force in wire drawing. He defines a quantity Z as the ratio of the drawing force to the force required to produce simple tensile yielding in the final cross section, and computes the minimum value of Z as a function of the drawing angle for various reductions and friction coefficients. Several curves and a table are given to illustrate the results.

P. G. Hodge, Jr., USA

3099. MacLellan, G. D. S., Some friction effects in wire drawing, *J. Inst. Metals* **81,** 1–13, 1952/1953.

Paper develops a generalized version of Sachs' theory of wire drawing to allow for the effect of a cylindrical extension to a conical die channel, commonly called a "parallel." Such a parallel, though undesirable, is unavoidable with certain methods of manufacture. Expressions are derived for back-pull factor (slope of the linear relation between axial load on die and back pull) and drawing stress in terms of wire and die dimensions, die angle, coefficient of friction, and yield strength of wire. Previously published results are re-examined in the light of this analysis, and reasonable agreement is found.

Results of further experimental investigations by the author are given. Tests utilized 12° and $5^1/_2$° diamond dies and 6° tungsten carbide die of ordinary industrial design, the test wire being a slightly hardened copper wire of about 0.05-in. diam. The length of parallel could not be measured directly, but magnitudes of parallels implied by theoretical analysis of test results were not unreasonable. If a direct measure of the length of parallel had been possible, it would have greatly enhanced the value of the paper. A device for accomplishing this apparently is now available for use in future investigations.

The test results show, however, that parallels effect an appreciable increase in drawing force, and that allowance must be made for their effect in the derivation of coefficients of friction from back-pull experiments. Values of coefficients of friction resulting from theoretical analysis of back-pull tests are mostly in the range 0.06–0.10. Allowance for parallel also provides an explanation of presence of initial intercepts on curves of drawing force plotted as a function of reduction of area, hitherto not satisfactorily explained.

Attempts to determine coefficients of friction from experiments with split dies were inconclusive, probably due to penetration of lubricant between halves of the split die. Further efforts in this direction are planned. W. E. Black, USA

3100. Finley, E. M., Bearing strengths of some 75S-T6 and 14S-T6 aluminum-alloy hand forgings, *NACA TN* 2883, 24 pp., Jan. 1953.

Results are given from an investigation of bearing properties of some 75S-T6 and 14S-T6 aluminum-alloy hand forgings in the longitudinal and long transverse directions and in surface and center locations. The tensile properties of the forgings were above those specified for such material and showed the expected directional or locational characteristics. The bearing properties showed less directional or locational variation than did the tensile properties. It was found that ratios of bearing to longitudinal tensile strengths for 75S-T6 forgings were slightly lower than those previously reported for sheet, but not quite so low as those observed for rolled and extruded bar in the 2- to 3-in. thickness range. The ratios for the 14S-T6 forgings were in fair agreement with those used in ANC-5 for hand forgings in the cross-sectional area group of 36 to 144 sq in. and having a forged length of not more than three times the width. Recommended nominal ratios of bearing to longitudinal tensile strength are given for use in selecting design bearing strengths. G. Gerard, USA

3101. Gilbert, H. L., Johansen, H. A., and Nelson, R. G., Forging of arc-melted chromium, *J. Metals* **5,** 1, 63–65, Jan. 1953.

High-purity electrolytic chromium plate has been hydrogen-reduced and arc-melted under inert atmosphere to give sound ingots. These ingots may be hot forged to break the as-cast structure and then worked at 500 C to finished form. An intermediate and final high-temperature anneal gives best ductility and grain refinement. From authors' summary

3102. Design of precision grinding machines, *Engineer, Lond.* **195,** 5062, 170–171, Jan. 1953.

3103. Rapatz, F., and Motalik, F., Slope and evaluation of tool-life cutting-speed curves (in German), *Stahl u. Eisen* **72,** 25, 1583–1587, Dec. 1952.

HSS tools were checked for long-time operations in turning, and it was found that tool-life cutting-speed curves do not always run straight for all cutting speeds, even when machining under identical conditions. For chips of different cross section, they are not parallel. Related investigation of metal cutting in Europe and the United States is also discussed. A. O. Schmidt, USA

3104. Opitz, H., and Kob, J., Cutting forces and temperatures when milling with carbide cutters, *Tool Engr.* **30,** 1, 35–41, Jan. 1953.

Studies are reported of the effect of the cutting speed, feed, and depth of cut on the tool forces and cutting temperatures

when SAE 1010, 1035, and 1060 steels are milled with carbide cutters. In general, the tangential cutting force has been found to decrease and the cutting temperature to increase as the cutting speed is increased. By changing the feed from 0.002 to 0.012 ipt, the tangential cutting force increases approximately linearly while the specific cutting force diminishes. Increase in feed also results in a higher cutting temperature. However, the latter increases less rapidly with an increase in feed than with an equally proportional increase in cutting speed. Variations in the depth of cut ranging from 0.040 to 0.360 in. do not produce any measurable effect on either the specific cutting force or the cutting temperature.

The manner by which the cutting edge of the carbide tool engages with the workpiece constitutes, in many cases, a decisive factor in tool life. Optimum rake angles for milling steel of tensile strength ranging from 50,000 to 140,000 psi have been established to lie within the limits of negative 7 to 13 deg. For medium and hard cast irons, the optimum rake lies between 0 and negative 5 deg.

Experimental results on turning steels carried out by the reviewer and his colleagues at the University of Illinois agree with a majority of the statements made by the authors.

B. T. Chao, USA

3105. **Oathout, J. D., Howell, W. L., Jr., Hamer, J. P., and Leland, H. L., A lathe test for the evaluation of cutting fluids,** ASME Ann. Meet., New York, Dec. 1952. Paper 52—A-49, 13 pp., 6 figs.

Test involves standardizing material tool and cutting conditions. Cutting speed vs. tool-life curves obtained with different fluids are compared with that for dry cutting. Results show correlation with industrial performance.

R. N. Arnold, Scotland

Hydraulics; Cavitation; Transport

(See also Revs. 3248, 3262, 3278)

3106. **Al'tshul', A. D., On the determination of the loss of head in canals and pipes** (in Russian), *Gidrotekh. Stroit.* no. 6, 44–47, June 1952.

An interesting discussion appeared in a Russian magazine of hydraulic engineering concerning the rational formula for pipe and open-channel flow. In 1949, I. I. Agroskin proposed a new formula based on investigations of irrigation and drainage channels in the United States and Central Asia: $C = 1/n + 17.72 \log R$ (for metric units), which would correspond, in English units, to $C = 1.486/n + 26.3 \log R - 13.60$. Here C is the Chezy formula factor, R the hydraulic radius, n the roughness coefficient, which can be correlated with the linear unevenness of the walls. That formula was suggested as the only reasonable one in the standard textbook on hydraulics by I. I. Agroskin, G. T. Dmitriev, and F. I. Pikalov, 1950.

Opponents (V. F. Poiarkov, G. I. Kasianov) disclosed many inconsistencies and errors in the development of the Agroskin formula, in which, for no reason, it is assumed that $C = 1/n$ at $R = 1$ m. Therefore, the Agroskin formula is merely an empirical one, particularly inaccurate at R less than 0.5 m and more than 3.5 m, even less accurate than the Manning formula or Pavlovskij formula with exponent of R varying with the roughness.

Al'tshul' in his discussion advocates his own formula [AMR **4**, Revs. **2971, 3283**], adapted for computation of pipes, $f = 0.1\,(k/d + 100/Re)^{0.25}$, where d is pipe diameter, Re the Reynolds number, k is so-called equivalent roughness, related to the linear unevenness Δ, approximately, $\Delta = 0.07\,k$. At large values of Re in comparison with 100, this formula becomes $f = 0.1(k/d)^{0.25}$, evidently constant for very rough pipes; while at small values of k/d, it approaches the familiar Blasius formula for very smooth pipes. Al'tshul' adapted his formula for water flow in open channels in the form $C = 20 \log vR/(v\epsilon + 10)$; here ϵ is a reduced linear roughness, different from Δ in pipes. Formulas were checked by the experimental data by Freeman, de Marchi, and several Russian authorities.

The main objection by the reviewer is that these "rational" formulas still are not dimensionless, as many "coefficients" have their own dimensions.

S. Kolupaila, USA

3107. **Rémenieras, G., Simple device for reducing the speed of elasticity waves in pipes; application to protection against certain water-hammer effects** (in French), *Houille blanche* **7**, no. A, 172–195, Mar. 1952.

The first part of paper summarizes the well-known theories of Allievi and of Gariel on the maximum pressure rise in conduits. It is then shown how this pressure rise depends on the pressure wave velocity a. If this is changed to a', this pressure rise will vary and the author indicates when a decrease of the velocity will produce a lower and when a higher pressure rise, and by which amount the pressure varies.

The second part investigates methods of producing a drop of the wave velocity, the only practical one consisting in an elastic (rubber) pipe filled with air lying inside the main conduit along its whole length. The third part of the paper summarizes the results of tests carried out at the French Hydraulic Research Laboratory of Chatou.

C. Jaeger, England

3108. **Dressler, R. F., and Pohle, F. V., Resistance effects on hydraulic instability,** *Comm. pure appl. Math.* **6**, 1, 93–96, Feb. 1953.

Paper is generalized derivation of criterion for formation of roll waves or slugs in open-channel flow. Dependence of friction term in unsteady-flow equation on velocity and depth is considered by including each to a general exponent. Resulting instability condition includes both exponents. Previously derived expressions have been dependent on Chezy's or Manning's equation. Final conclusion is that both varying depth and velocity must be included for correct criterion.

W. D. Baines, Canada

3109. **Winkel, R., Technical possibilities of guiding a swift current into slowly flowing water with a minimum of losses** (translated by J. C. van Tienhoven from *Bautechnik* **28**, 12, 1951), *Wuys. Exp. Sta. Transl.* no. 53–2, 5 pp., Mar. 1953.

For an expanding transition in free surface flow, author cautions use of the head loss which may be derived by use of the momentum and energy principles over sections of uniform flow upstream and downstream. Derivation does not account for the shape of the transition. A gradual transition eliminates the head loss resulting from the large eddies in the zone of separation of an abrupt transition.

H. G. Farmer, Jr., USA

3110. **Halbronn, G., Study of the establishment of the flow regime on high gradient structures. Application to air-entrainment problems** (in French), *Houille blanche* **7**, 1, 3, 5; 21–40, 347–371, 702–722; Jan.-Feb., May-June, Oct.-Nov. 1952.

This treatise deals with the flow on long overflow weirs when a uniform regime tends to be established on the spillway apron due to the effects of water viscosity. The boundary layer forming near the crest extends itself along the spillway apron, passes from laminar to turbulent and reaches a stage at which it occupies the entire height of the flow at a point named by the author "the criti-

cal point." From this mean position the free surface becomes rough and, if the transversal kinetic energy given to the superficial particles by the turbulence is sufficient to overcome capillarity effects, the water clouds and becomes "white," indicative of an entrainment of air in the center of the flow mass.

In the first part of this thesis, author establishes theoretically the general equations governing the evolution of the boundary layer on a spillway apron of infinite width, taking as a basis the von Kármán relation established for pipes. The simple systems of relations which are evolved make possible a detailed knowledge of the flow upstream from the critical point and even of the flow downstream if white water does not appear. The resolution of these systems enables one to see quickly whether the critical point appears on the spillway apron in an overflow weir project.

The course of the calculation is followed several times in the second part of the article when the author applies the results of his calculations to tests made by him and others. The reconciliation is satisfactory as long as white water does not appear on the weir.

When white water appears downstream from the critical point, the presence of air in the mass of water modifies the distribution of speed in the cross sections of the flow, which tend toward a uniform regime with the entrainment of air. The calculations developed in the first part no longer apply downstream from the critical point. Nevertheless, it is necessary to have a criterion indicating whether white water may appear on a weir project, in which case the determination of the geometrical characteristics of the installation will have to take into account the entrainment of air. It is this that the author attempts to elaborate in the third part of his study. After having specified the causes which produce the phenomenon and the way in which they act, author proposes a criterion of the instability of the free surface. Although this criterion is not rigorous and has not been submitted to any experimental control, it is, nevertheless, an interesting working aid, making it possible to discover whether a special problem requiring study on a model is presented in a projected installation.

Generally speaking, this study constitutes a remarkable tool for the design engineer.

It should be remembered that a similar study was made later at Iowa Institute by W. J. Bauer, in which he experimented rough and smooth bottoms inclined to 20, 40 and 60°, and, by so doing, he generalized Halbronn's experimental work.

G. Cohen de Lara, France

3111. Horváth, S., Considerations on the preparation of a comprehensive plan for the Danube regulation (in Hungarian, with Russian, French, and German summaries), *Vízügyi Közlemények* no. 1, 3–20, (4)–(13), 1952.

Importance of paper is evident in view of the fact that the Danube is the only transcontinental river in Europe, eight nations occupying its shores. Introductory part of this lengthy paper discusses the necessity of preparing a scheme, and lists the present existing difficulties. Frequency and duration curves are given for low water shoals and it is shown that a 700-ton total capacity barge might have to operate at reduced loading for approximately half of the year on the Hungarian-Czechoslovakian stretch. The effect of finite depth on the drag is analyzed, giving results for a barge (approximate dimensions: length 200 ft, beam 27 ft, draft 5 ft, displacement 613 tons). It is shown that the drag at 6.5 kts is doubled if the depth changes from approximately 14 ft to 6.5 ft and tripled at 6 ft.

The main body of the paper discusses the following questions: Connection with and relation to other rivers; correlation between navigational and other uses; and permissible minimum dimensions of the cross section. Suggested preliminary work: Establish a unified international minimum depth; find best, generally suitable ship type; organize systematic level observation; investigate ice and sedimentation problems: and organize international conference.

This paper contains many repetitions, political phraseology of little technical interest, and no new results. The few useful data and the recognition and listing of economic-technical problems reward the reader's patience. (Reference not mentioned in paper but suggested by reviewer: "Waterway engineering" by O. Franzius, 1936.) V. G. Szebehely, USA

3112. Kovács, G., Hydromechanical comparison of methods recommended for calculation of backwater curves (in Hungarian, with Russian and French summaries), *Vízügyi Közlemények*, no. 1, 81–109, (26)–(27), 1952.

In the first part of the paper, author derives the basic differential equation for backwater curves in uniform channels, making use of Chezy's formula to include friction. As a starting point, various forms of the Bernoulli equation are used which author associates with Rühlman (1880), Tolkmitt (1907), Bakhmeteff (1932), and recent Russian writers, Pavlovsky, Rachmanov, and Bernadsky. (Reviewer notes that author omits J. A. C. Bresse's fundamental work "Méchanique appliquée," 1860). It is shown that the afore-mentioned contributions reduce to the same final form. (Reviewer notes that this form is included in standard texts in this country; see, e.g., "Fluid mechanics for hydraulic engineers" by Hunter Rouse, p. 290, Equ. 215.)

In the second part, neglecting Mononbe's [*Proc. Amer. Soc. civ. Engrs.* **62**, 5, 643, 1936] and reviewing von Seggern's work [AMR **2**, Rev. 651], author proceeds to show that when the effect of velocity head changes can be neglected, the basic equation reduces to $\Delta z = Q^2(\Delta l/K^2)$, where Δz is the change of water surface with respect to the horizontal, Q the discharge, Δl the length of step, $K = cF(R)^{1/2}$ is the conveyance of the channel, c is Chezy's constant, F cross-sectional area, and R hydraulic radius. With a somewhat doubtful analysis, author shows that the above approximation—which this reviewer recognizes as the Chezy formula but author attributes to Russian authors—results in a maximum error of 3.8% in the water-surface slope for practically important cases. Final part of the paper discusses Pavlovsky's method (the conveyance is computed within steps by means of graphical differentiation) and converts it to a purely analytical method. In conclusion, author cites examples wherein application is made to surface profiles of the two greatest rivers of Hungary, the Danube and Tisza.

These examples appear to be the only new contributions to the literature. V. G. Szebehely, USA

3113. Levy, I. I., Norms and technical specifications adopted in the U.S.S.R. for designing settling basins, *Trans. Fourth Congr. Inter. Comm. Large Dams*, New Delhi, Jan. 1951, in 4 vols., vol. IV, pp. 525–541. $48 per set.

The theory presented was developed by Prof. M. A. Velikanov in connection with the development of settling basins to clarify water diverted from streams into hydroelectric plants. In the cases discussed by the author the objective was to settle out all sand coarser than about $^1/_4$ mm.

The theory is based on the following assumptions: (1) The flow velocity is uniform throughout the basin. (2) The sediment concentration in the flow at the inlet to the basin is uniform over the depth and width. (3) Sediment entering the basin through an elementary area at a distance h above the floor of the basin is spread upward and downward by turbulence as it is carried downstream with the uniform velocity U of the flow. (4) The dis-

tribution of concentration of sediment which enters at a level h assumes a normal distribution with standard deviation σ as it is carried through the basin, and the position of the mean settles with a uniform velocity w, where w is the terminal settling velocity of the individual sand particles in still water. (5) Any sediment which in an imaginary basin of unlimited depth would settle below the level of the bottom of the real basin is permanently removed from the flow.

The value of σ was determined from two series of laboratory experiments and one series of field experiments and was found to be $\sigma = (sh/2)^{1/2}/2.73$ where s is the horizontal distance from the basin inlet. With this value of σ and the above assumptions, the percent W of the total sediment entering a basin that is removed is calculated. The results are given in a chart of W at a function H/S for a wide range of the ratio W/U, where H and S are the depth and length of the basin, respectively.

Taking the relations between σ and the diffusion coefficient ϵ according to Taylor's theory, $\epsilon = U d\sigma^2/ds$, and, comparing the author's theory with that of Camp, we find good agreement.

V. A. Vanoni, USA

Incompressible Flow: Laminar; Viscous

(See also Revs. 3106, 3127, 3129, 3136, 3186, 3203, 3260, 3261)

3114. Gorup, G. v., New method for calculation of stream functions for flow patterns of time-dependent contour (in German), *ZAMM* **32**, 11/12, 371–378, Nov./Dec. 1952.

Work is concerned with finding the stream functions corresponding to the flow of ideal fluid surrounding a cylinder of infinite length and a time-variable contour. Under assumption of a meromorphic mapping function, the stream function is determined by the boundary condition on the contour and the condition of regularity. To express the boundary condition, author introduces a functional equation and shows how solutions of this equation satisfying the condition of regularity can be obtained. The method has particularly simple features in the case of a rigid contour. The application of the method is illustrated by an example treating the Joukowsky airfoil.

J. Beránek, Czechoslovakia

3115. Karal, F. C., The motion of a sphere moving parallel to a plane boundary, *J. appl. Phys.* **24**, 2, 147–151, Feb. 1953.

The motion of a rigid sphere moving through an incompressible, homogeneous, nonviscous fluid in the presence of a rigid, infinite plane boundary is examined. The direction of motion of the sphere is assumed to be parallel to the plane boundary. It is assumed that the motion is irrotational and that the sphere velocity is small. An exact treatment is presented. Several curves of the pressure variation on the plane boundary are plotted for different values of the ratio of sphere radius to the perpendicular distance from the center of the sphere to the plane boundary.

From author's summary by G. M. Lilley, England

3116. Chandrasekhar, S., The stability of viscous flow between rotating cylinders in the presence of a magnetic field, *Proc. roy. Soc. Lond.* (*A*) **216**, 1126, 293–309, Feb. 1953.

Theory is developed for calculating neutral stability criterion for laminar viscous flow of electrically conducting incompressible fluid between corotating cylinders in the presence of a magnetic field. Basic equations are solved in detail, and numerical results presented for case when the two cylinders differ in diameter by an amount small compared to mean diameter. Effect of conducting media on critical Taylor number is determined by a single dimensionless parameter involving field strength, fluid density, cylinder spacing, and coefficients of electrical conductivity, kinematic viscosity, and magnetic permeability. Similar analysis for more complicated case of counter-rotating cylinders is promised for a later paper.

D. R. Chapman, USA

3117. Rouse, H., and Hsu, H.-c., On the growth and decay of a vortex filament, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 741–746, 1952.

Authors consider the growth and decay of a combined vortex in a viscous fluid starting with Goldstein's solution for the velocity field [*Proc. Lond. math. Soc.* **34**, s.2, 1932]. For the limiting case of zero core diameter, their results agree with those of Bateman [*NACA Rep.* 144, 1922 (not referenced)]. Expressions are obtained which describe the motion subsequent to elimination of the generating vortex filament and which involve three reference parameters related to characteristics of a vortex at a given instant—in their case, the maximum circulation, the initial core radius, and the centerline vorticity.

P. Eisenberg, USA

3118. Givellato, S., Determination of the induced velocity due to a system of p helicoidal vortexes of sinusoidally varying strength and an associated vortex system (in Italian), *Monogr. Lab. aeron. Politecn. Torino* no. 301, 22 pp., 1952 = *Atti Accad. Sci. Torino* **86**, 1951–1952.

Aim of study is the determination of the induced velocity due to a system of p vortexes, each of which consists of a helicoidal vortex of sinusoidally varying strength, of a vortex situated along the axis of the helix, and of a system of radial vortexes connecting the axis with the helix. A helicoidal vorticity surface is thus produced. The total potential corresponding to the system is deduced by means of series developments. It was found advantageous to introduce an auxiliary potential $(\Phi)_{r=\infty}$ due to the above-defined vorticity system, provided the radius of the helix becomes infinite. In order to express this potential, the functions $T_m(x)$ are introduced and several properties of them are studied. Finally, mean values for the induced momentum and the moment of momentum are calculated.

From author's summary by M. A. Dengler, USA

3119. Jaeckel, K., Simplified derivation of the Weissinger circulation equation for the sweptback wing (in German), *ZAMM* **33**, 1/2, 65–66, Jan. 1953.

Author derives Weissinger circulation equation for sweptback wings without introducing a source flow; instead, he starts from a direct calculation of the induced velocity of the trailing vortex sheet.

Maurice Holt, England

3120. Hamel, G., Spiral motions of viscous fluids, *NACA TM* 1342, 44 pp., Jan. 1953.

Translation from *Jahresber. d. deutschen Math. Ver.* **25**, 1917.

3121. Douglas, O., A series of low-drag aerofoils embodying a new camber-line, *Aero. Res. Counc. Lond. Rep. Mem.* 2494, 19 pp., 1947, published 1952.

Problem is to find a simple usable stock-type camber line from which the fairing of an airfoil is, in each case, designed specifically to suit the camber. Camber line should be simple in use and readily adapted to meet different design requirements.

Author derives a camber line that fulfills the above and is analytically defined (two sine terms of a Fourier series), yet is adjustable for any particular case by choice of optimum lift coefficient.

An example design of an airfoil is given in detail. Also pre-

sented is a series of low-drag airfoils and a favorable comparison with a corresponding rooftop airfoil. Conclusions are that the use of this camber line has distinct advantages over the "constant-load" type; namely, it is possible to obtain, among other things, lower values of pitching moment, larger lift-coefficient range, and larger values of critical Mach number. J. DeYoung, USA

3122. Yih, C.-S., Diffusion from a line source in laminar flow over a wedge and in Blasius flow, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 797–800, 1952.

In symmetric flow over a wedge or plate, a change in the fluid, due to some source at the leading edge, is considered. Free convection is neglected; velocity distribution is assumed to be unaffected by the source.

A general diffusion formula is deduced from the differential equation of Falkner and Skan for the boundary layer of a wedge, based on the diffusion equation, dimensional considerations, and appropriate boundary and continuity conditions. For the special case of flow along a plate, elaborate numerical results are given.
H. J. Schoemaker, Holland

3123. Nadile, A., Ellipsoidal configurations of the equilibrium of a liquid and a homogeneous mass attracted by a concentric circular ring (in Italian), *Atti Semin. Mat. Fis. Univ. Modena* **5**, 178–189, 1950–1951.

Author proves the existence of the configurations in the title (he supposes the ring in a dimetral plane and very far) when the components of the velocity of a particle are a linear function of its coordinates. He considers also the particular case of rotating masses of liquids. D. Graffi, Italy

3124. Woronetz, C., Influence of gravity on the form of liquid jets (in French), *C. R. Acad. Sci. Paris* **236**, 3, 271–273, Jan. 1953.

Change in shape of a liquid jet emerging from a slit due to gravity is calculated, using extension of formula due to Demtchenko and assuming that the slit width is small compared with the height of liquid in the container. J. A. Lewis, USA

3125. Platzman, G. W., Starr's invariant, *Tellus* **4**, 4, 352–355, Nov. 1952.

In 1950, R. Fjortoft published a paper on the application of integral theorems in deriving criteria of stability for laminar flows and for the baroclinic circular vortex [*Geof. Publ.* **XVII**, no. 6]. In chap. II of this paper he considers, among others, the case of a homogeneous fluid enclosed between fixed boundaries (e.g., two latitude circles). In a note in *Tellus* [**2**, no. 4, p. 321, 1950], V. P. Starr calls attention to Fjortoft's paper and shows that, in the case mentioned above, Fjortoft's result may be written in an especially clear form by using Starr's quasi-Lagrangian system of equations. He arrives at an integral expression which is invariant in the case treated.

The present author shows that Starr's integral can be used under more general assumptions about the distribution of vorticity than stated by Starr. He also discusses the meaning of the conception of stability as involved in the two papers quoted above. H. Merbt, Sweden

3126. Tollenaar, D., Exchange of liquid between paper pores, *Appl. sci. Res.* (A) **3**, 6, 451–461, 1953.

The assumption of a Gaussian distribution of capillary radii, requiring three constants (average radius, spread of radii, and capillary density), was fitted by trial and error to data on capillary rise vs. liquid volume in paper. Good agreement was reached with eight experimental points, as well as with an independent measurement of capillary surface by methane adsorption. It is shown that the water travels upward primarily in the large capillaries, but is bled from them and retained initially by the small ones. C. F. Bonilla, USA

Compressible Flow, Gas Dynamics

(See also Revs. 3171, 3176, 3188, 3190, 3192, 3204, 3207)

3127. Milne-Thomson, L. M., Theoretical aerodynamics, 2nd ed., New York, London, Toronto; D. Van Nostrand Co., Inc., 1952, xviii + 414 pp. $10.

The second edition is identical with the first except for the addition of a chapter on linearized supersonic conical-flow theory. The book is an excellent presentation of topics from incompressible nonviscous fluid theory which are useful in aerodynamics. Selected topics from compressible-flow theory are included.

A very detailed presentation is given of two-dimensional incompressible flow, using the complex potential and conformal mapping and leading to the flow about airfoil profiles of various shapes. A chapter on thin airfoil theory is also presented. Vortex laws are discussed leading to the theory of wings of finite aspect ratio, and lifting-line theory is discussed in detail. There are short chapters on incompressible lifting-surface theory, propellers, and effect of wind-tunnel walls. A sketchy introductory chapter to compressible flow is provided, with emphasis on two-dimensional linearized supersonic flow. The discussion of three-dimensional supersonic-wing theory is restricted to linearized conical flow. A complete discussion is given, following Busemann and Lagerstrom, of the solution of the elementary planar potential problems of conical flow by the use of complex variables and analytic continuation. No application of conical fields to the construction of more complicated fields is made. The book is completed with brief chapters on the dynamics and stability of conventional aircraft in flight, and on vector analysis.

The reviewer believes that in the classical part of the book the mathematical presentation is excellent, with the author carefully and clearly explaining the theory and even points which are usually overlooked; e.g., leading-edge singularities. The chapter on compressible flow theory is too sketchy; it omits mentioning the throat at sonic velocity, oblique shock waves, transonic flow, relationship to acoustics. Treating supersonic-wing theory wholly by conical flow is elegant but not the clearest method physically or logically. Reviewer believes discussion of the theory based on surface distribution of sources and vortexes is to be preferred. Some significant omissions in the book are nonsteady motion and flow about bodies. Within the limitations just mentioned, this book is an excellent treatise featuring careful working out of many examples and containing many exercises. For a critical discussion of underlying hypotheses and comparison with experiments, the reader has to look elsewhere. J. D. Cole, USA

3128. Graffi, D., Uniqueness theorem in dynamics of compressible fluids (in Italian), *J. rational Mech. Analysis* **2**, 1, 99–106, 1953.

The author takes up the remarks of Birkhoff in his recent book ["Hydrodynamics, etc.," Princeton, 1950] in regard to the uniqueness of solution of the Navier-Poisson equations for viscous fluids. After calling attention to the simple method used by Foà in 1929 to prove uniqueness for the incompressible case under broader conditions than those used in the more elaborate analytical works cited by Birkhoff, the author proceeds to formulate and prove a corresponding theorem for compressible fluids.

whether viscous or not. The precise statement would be too long here. The most important physical restrictions are that the flow shall be barotropic, the viscosities independent of the density, and the density uniformly bounded below by a nonzero constant. Shock waves are excluded, but higher-order waves (e.g., finite sound waves) are not. The most important boundary-value problem included is that of flow about an obstacle; in this case, the velocity and density are supposed to possess limit values at infinity, their difference from which is $o(r^{-3/2})$, while, in the case of a viscous fluid, the velocity gradient is assumed $o(r^{-1/2})$.

Under these and other precisely formulated assumptions, the author proves that there can be at most one solution of the general initial value problem, for all time.

The proof begins with the subtraction process used for linear theories. Various parts of the nonlinear remainder are manipulated into forms where they can be bounded by simple expressions. After a number of ingenious estimates the author finally gets a non-negative integral over the whole flow, whose time derivative is bounded by another non-negative integral. Integrating this over an arbitrary time interval, he is able finally to show that the volume integral of the sum of the squares of the velocity difference and the density difference is bounded by its own time integral. Hence this volume integral must vanish, and hence so must the two differences.

The author's result is the strongest uniqueness theorem for fluid dynamics which has been obtained up to now. It would be wrong, however, to regard the subject as closed. First, the assumption of barotropic flow, which allows the author to avoid using the energy equation, is too restrictive. Second, the author's analytical assumptions, while certainly not known to be wrong, are extremely severe. In particular, their uniformity in *time* is a drastic requirement. We have at present no indication that any solutions satisfying them exist. A full analytical theory would have to investigate the lower bound for the density, the order conditions at ∞, and the smoothness assumption.

C. Truesdell, USA

3129. **Garrick, I. E., On moving sources in nonsteady aerodynamics and in Kirchhoff's formula,** Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 733–739, 1952.

Kirchhoff's formula, the mathematical formulation of Huygens' principle for sound waves in air, provides a framework for a generic treatment of the role of sources and doublets in linearized nonsteady aerodynamics. It may be extended to apply to a uniformly moving medium with the aid of solutions for moving sources and doublets of arbitrary strength. Formulas for these solutions for subsonic, sonic, and supersonic speeds, for space and for the plane, are discussed and some geometric properties are shown. Some illustrative results on two-dimensional oscillatory air forces for a range of transonic speeds, including Mach number 1, are presented. Other applications to transient flows and to the pressure field of a rotating propeller are indicated.

From author's summary by M. J. Thompson, USA

3130. **Sedov, L. I., On the general theory of one-dimensional gas motions** (in Russian), *Dokladi Akad. Nauk SSSR* (*N.S.*) **85,** 4, 723–726, Aug. 1952.

Relations are derived which the author suggests may lead to approximate solutions or asymptotic results for unsteady flow of ideal gas in one dimension or with cylindrical or spherical symmetry. First, conditions near the origin are examined by expanding all flow quantities in powers of the radius, with coefficients depending upon time. It is shown how all coefficients can be found in terms of initial conditions and (say) the time variation of density at the origin. Second, expressions are given for flow gradients just inside a moving outer boundary, including the case of a shock wave moving into still fluid. Finally, interpolation between these extremes is suggested, and the influence of viscosity and heat conduction briefly discussed.

M. D. Van Dyke, USA

3131. **Robinson, A., Non-uniform supersonic flow,** *Quart. appl. Math.* **10,** 4, 307–319, Jan. 1953.

Paper treats two-dimensional problem of accelerated (and decelerated) supersonic flow. The usual linearization of the equations of motion is employed for irrotational flow of a compressible, isentropic gas. The boundary condition at the airfoil boundary is satisfied on the axis (chord line). Acceleration of the main flow is given in terms of the variation of static pressure. For the particular case of linear variation of main stream pressure, a closed analytical expression is given for the pressure distribution around an airfoil. Simplified approximations are also given which appear valid over about the same range as any linearized theory will permit, i.e., transonic and hypersonic regimes are excluded. The lift and wave drag coefficients of a thin airfoil (flat plate) and a symmetrical double wedge are calculated from the equations derived.

H. P. Liepman, USA

3132. **Miles, J. W., On linearized theory,** *J. aero. Sci.* **20,** 1, p. 64, Jan. 1953.

In this short note, necessary conditions are stated for linearization of equations for unstationary compressible irrotational three-dimensional flow past a thin and slender body (wing of large span), introducing simultaneously three transonic parameters, referring to Mach number, wing aspect ratio, and reduced frequency together with wing thickness. Each of these parameters has, separately, already been introduced by former authors on the subject. A short discussion on sufficiency of the conditions terminates the note.

R. Timman, Holland

3133. **Ferrari, C., On the potential of a vortex line in linearized supersonic flow** (in Italian), *Monogr. Lab. Aero. Politecn. Torino* no. 287 = *Atti Accad. Sci. Torino* **86,** 6 pp., 1951–1952.

Using a formula established by A. Robinson [AMR **2,** Rev. 1293] for the velocity induced by a vortex distribution in a linearized supersonic stream flow, author finds an expression for the potential of the velocity induced by a vortex filament in the form of a potential due to a doublet distribution over an arbitrary open surface which has the vortex filament as boundary. The doublet density is proportional to the circulation of the vortex filament, and the orientation is in the direction of the co-normal.

P. Chiarulli, USA

3134. **Ribner, H. S., Convection of a pattern of vorticity through a shock wave,** *NACA TN* 2864, 48 pp., Jan. 1953.

Paper deals with weak turbulence (idealized as plane sinusoidal shear wave) and shock waves. Reference frame is fixed with respect to moving disturbance pattern along shock wave, thus giving a steady-flow problem for an oblique shock wave. Linearized equation for disturbance stream function downstream is derived, using relation between vorticity (shear wave) and enthalpy and entropy gradient, assuming main velocity to be parallel. Boundary conditions behind shock wave are obtained from well-known shock-wave relations. Nonhomogeneous equation for stream function is solved by assuming a sinusoidal wave pattern and adding a similar wave pattern that satisfies the homogeneous equation. Unknown coefficients of these solutions (and

in boundary conditions caused by unknown perturbations of shock-wave shape) can be uniquely determined. Solution of non-homogeneous equation is shown to correspond to shear wave (vorticity), solution of homogeneous equation corresponds to potential flow (pressure wave). For oblique shock (with respect to incoming wave), a critical angle of incidence exists, separating subsonic flow downstream from supersonic flow. In the former case, pressure wave damps out; in the latter case, it is propagated as ordinary Mach wave.

In a numerical example, the sound wave is shown to have large intensity (100 db). R. Timman, Holland

3135. Cunsolo, D., Calculation of the shape of a two-dimensional supersonic nozzle in closed form, *NACA TM* 1358, 29 pp., Jan. 1953.

See AMR **5**, Rev. 1524.

3136. Adams, M. C., and Sears, W. R., Slender-body theory—review and extension, *J. aero. Sci.* **20**, 2, 85–98, Feb. 1953.

Authors define slenderness, review Munk's and Jones' slender-body theories, and correlate them with the results of von Kármán's Volta Congress paper, referring to Ward [AMR **3**, Rev. 529]. Ward's theory, based on a Laplace transform of the complete Prandtl-Glauert equation, is applicable to smooth pointed bodies of arbitrary cross section, camber, and incidence in supersonic flight. Authors extend Ward's work to the same types of bodies in subsonic flight, using a Fourier transform. Discrepancies in the existing theories for pressure, lift, supersonic wave, and induced drag are explained, also for unsteady flow where "indicial," i.e., transient problems must be distinguished from oscillations. The slender-body theory is then extended to a new theory of "not-so-slender" bodies by expanding the velocity potential of linearized theory in terms of a slenderness parameter and retaining only the largest terms; this parameter is defined as $(1 - M^2)(s^2/l^2)$ where s is a characteristic width and l the over-all length. In order to make this idea useful, line singularities are distributed not only on the axis but also at various spanwise locations. The method is carried out for the subsonic case (Fourier transform) and the supersonic case (Laplace transform), and the integral equations are established.

Applications of this second-order theory are shown for a flat delta wing at incidence and for the chordwise lift loading of a flat plate; applications so far are restricted to supersonic flow. The drag of a supersonic wing with zero thickness, including leading edge suction, can be obtained to second order. Supersonic wings of minimum drag are discussed and, for special cases, results are compared with those of first-order and slender-wing theories.

G. R. Graetzer, USA

3137. Robinson, A., Aerofoil theory of a flat delta wing at supersonic speeds, *Aero. Res. Counc. Lond. Rep. Mem.* 2548, 21 pp., 1946, published 1952.

Paper presents details of analysis omitted in 1946 report. It features separation of variables in hyperboloido-conal coordinates for both sub- and supersonic leading edges. Results were obtained independently in U.S. and elsewhere at the same time (Snow, Lagerstrom, Stewart, Bartels, Germain, etc.).

A. Charnes, USA

3138. Pistolesi, E., The drag of a quasi-cylindrical tube in a supersonic flow (in Italian), *Aerotecnica* **33**, 1, 100–105, Feb. 1953.

The method of singularities distributed along the axis of a body of revolution, due to von Kármán, is applied to supersonic linearized flow past open cylindrical and quasi-cylindrical tubes. Given the shape of the tube and the angle of attack, one has to determine the intensity $f(\xi)$ of the singularities by means of an integral equation. Numerical solution is obtained by approximating the unknown function $f(\xi)$ not only by rectilinear segments $f(\xi) = k(x_0 - \xi)$, but also by a parabola arc $f(\xi) = -2k(\xi)$. Previous result of Brown and Parker [AMR **2**, Rev. 351] is mentioned as a special case. R. Sauer, Germany

3139. Jones, R. T., Theoretical determination of the minimum drag of airfoils at supersonic speeds, *J. aero. Sci.* **19**, 12, 813–822, Dec. 1952.

The induced drag of a thin wing in inviscid supersonic flow depends on the actual distribution of lifting pressure over the surface of the wing. In linearized theory, thickness drag may be computed separately and the two drag components added in the appropriate combination to represent a given wing. Criteria for the attainment of a minimum for each component of drag under certain fixed conditions were obtained by Jones in a previous paper [AMR **4**, Rev. 3334] through the use of reverse-flow theories. Criterion for wing of given planform and total lift is that superposition of induced disturbance fields in forward and reverse motions results in constant downwash over surface of wing. Similar conditions governing distribution of induced velocities over wing in "combined" flow field result from problems of how to distribute thickness for minimum drag, given maximum thickness, or given total volume of wing.

The determination of "combined" flow fields required for application of afore-mentioned criteria is complicated by the fact that in such fields the region of influence of wing extends both fore and aft of it, while usual forms of the elementary solutions, e.g., supersonic source, are characterized by a conical zone of action on one side of the source only and outside of this a zone of silence. This paper describes a method for determining combined disturbance fields based on the representation of elementary solutions by contour integrals, following the form introduced by Whittaker; such solutions have a zone of action extending in both forward and reversed directions. The distribution of sources or dipoles over the wing surface is then represented by a triple integral, in which the surface integral, after a change in the order of integration, represents a distribution of two-dimensional disturbances over the surface. Thus the three-dimensional flow is obtained, finally, by the superposition of elementary two-dimensional flows. This ingenious method simplifies the calculation of three-dimensional wing flows satisfying conditions for minimum drag.

Method is illustrated by example of wing of elliptic planform. Results are that induced drag is minimum when lift is distributed uniformly over surface. Optimum thickness distribution for minimum wave drag is wedge section for given frontal area and biconvex circular-arc type for symmetrical section with given volume. (These results are same as for two-dimensional airfoil in supersonic flow.) The minimum induced drag coefficient approaches the value $C_L^2/\pi A(M^2 - 1)^{1/2}$ for small aspect ratio A, and for large A approaches the two-dimensional value for a flat plate, $C_D = C_L^2(M^2 - 1)^{1/2}/4$. J. S. Isenberg, USA

3140. Tierney, J. A., An approximation to transonic flow of a polytropic gas, *Amer. J. Math.* **75**, 1, 43–56, Jan. 1953.

Starting with complete equation of inviscid irrotational gas motion in form $\psi_{\sigma\sigma} + K(\sigma)\psi_{\theta\theta} = 0$, where ψ is stream function, θ direction angle of streamline, and σ a complicated function of velocity, author discusses some details regarding approximation $K(\sigma) = \sigma - b\sigma^2$. For the constant $b = 0$, this reduces to Tricomi's equation. For $b = 2^{-1}(2\gamma + 5)(\gamma + 1)^{-1/2}$, where γ is adiabatic exponent, contact to third order in (p, ρ)-plane is ob-

tained at sonic point. An example is discussed using quadratic approximation to $K(\sigma)$. D. R. Chapman, USA

3141. **Harder, K. C., and Klunker, E. B., An application of the method of characteristics to two-dimensional transonic flows with detached shock waves,** *NACA TN* 2910, 16 pp., Mar. 1953.

The surface pressure distributions for a class of two-dimensional airfoils of fixed nose shape and arbitrary rear portion are obtained in near-sonic flow, once the surface pressure distribution is given for one member of the class. The latter information is sufficient to fix the position of the limiting characteristic (the last expansion wave to intersect the sonic line) on the airfoil. Simple wave flow is used to determine the pressure distribution downstream of the limiting characteristic. The error produced by neglecting the reflected compression waves from the sonic line is stated to be small, although detailed comparisons with exact theory are not given.

Nonlinear force characteristics in near-sonic flow are shown to be due to the extreme sensitivity of such flows to small disturbances, e.g., flap deflections. G. M. Lilley, England

3142. **Yoshihara, H., On the flow over a cone-cylinder body at Mach number one,** *Wright Air Development Center tech. Rep.* 52-295, vi + 15 pp., 8 figs., Nov. 1952.

Author describes the numerical scheme used to determine the flow. The sonic line which extends from the shoulder of the body to infinity divides the field of flow into subsonic and supersonic regions. First, the location of the sonic line is assumed and then the flow in each part is calculated independently. For the subsonic region, the relaxation technique is used, while flow in the supersonic region is determined by the method of characteristics. In general, the assumed position of the sonic line will not be correct and the subsonic and supersonic solutions will not match. A method of successive approximation is then introduced and the location of the line changed until the solutions each side of it are compatible. Author states the computational work involved is particularly heavy and that convergence may be slow.

The results obtained confirm the validity of the basic assumptions made by von Kármán and by Oswatitsch and Verndt in deriving their transonic similarity laws for slender bodies.

W. P. Jones, England

3143. **Fettis, H. E., An approximate method for the calculation of non-stationary air forces at subsonic speeds,** Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 723–732, 1952.

Author derives an approximate equation which has an exact solution and which is such that the kernel of the approximate equation may be brought as close as desired to that of the exact equation. From author's summary by A. Petroff, USA

3144. **Bader, W., The overflow cross section of cyclones** (in German), *ZAMM* **33,** 1/2, 66–67, Jan./Feb. 1953.

Brief mathematical note on frictionless flow conditions in a cyclone, assuming isentropic expansion of gas at entrance. Formula for rate of spin of gas core in terms of cyclone dimensions and flow rate is developed. No experimental data testing the validity of the result are presented.

J. M. DallaValle, USA

3145. **Cotter, J. R., Conduction of heat in a monatomic gas,** *Proc. roy. Irish Acad.* (*A*) **55,** 1–28, 1952.

Author treats the Boltzmann equation for elastic spheres for the problem of a one-dimensional steady heat flow. The simplifications appropriate to steady flow and one space variable are introduced, and the Boltzmann equation is linearized corresponding to small values of the heat flow. The resulting integral equation is identical with the usual one (Fredholm type) occurring in the first Chapman-Enskog approximation. A new method of solution of this integral equation is presented in which the equation is first transformed into a second-order ordinary differential equation and then integrated by an ingenious method yielding an alternating series and, therefore, simple error estimates.

Courtesy of Mathematical Reviews H. Grad, USA

Turbulence, Boundary Layer, etc.

(See also Revs. 3117, 3127, 3187, 3198)

3146. **Lees, L., On the boundary-layer equations in hypersonic flow and their approximate solutions,** *J. aero. Sci.* **20,** 2, 143–145, Feb. 1953.

Solutions, with interaction between leading-edge shock and laminar boundary considered, are obtained for both insulated and noninsulated flat plates. Skin friction and heat-transfer coefficients are shown as functions of Reynolds number for several Mach numbers up to 30. Coefficients increase rapidly with Mach number, the increase being more rapid the smaller the Reynolds number. Statement that streamline slope at boundary-layer edge equals slope of full boundary-layer thickness is not clear to reviewer. It seems that, by continuity equation, streamline slope at boundary-layer edge equals slope of displacement thickness—this only when product of density and velocity at outer edge of boundary layer is independent of position along plate.

N. Tetervin, USA

3147. **Emmons, H. W., and Bryson, A. E., The laminar-turbulent transition in a boundary layer,** Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 859–868, 1952.

Paper is part II of a study of boundary-layer transition phenomena which authors have observed to be a stochastic process. This paper is difficult to read in the absence of part I which was not published at time of review. Authors restrict treatment to applications, offering no hydrodynamic analysis but applying concepts of probability theory to correlate certain measurements on transition drag and heat transfer on a flat plate. Transition on a plate with swept leading edge is discussed as well as for a cone. Hypothesis for these calculations is that transition occurs in discrete spots which have a random spatial distribution. Authors argue that phenomena like the variation of transition Reynolds number (on a flat plate) with free-stream turbulence can be interpreted by relating turbulent intensity to the spot rate density g. The function g is connected to the fraction of time $f(P)$ that a given point P is turbulent by

$$f(P) = 1 - e^{-\int_R g\,dv}$$

where R is the "dependence volume" for the point P. For most applications, g is taken to be a constant independent of position.

Fig. 6 may lead some to understand that the free-stream turbulence was determined in Gebers' and in Schoenherr's flat-plate tests. Those interested in transition phenomena in water should consult *Taylor Model Basin Reps.* 724 [AMR **4,** Rev. 1685] and 726 [Sept. 1950] by M. Macovsky and J. Breslin, where it is shown from experiment that, at the onset of transition at fixed points, the flow is alternately laminar and transitional. These findings at least do not contradict the observations made at Harvard by the authors who could see the entire flow field at once.

J. P. Breslin, USA

3148. Nonweiler, T., The laminar boundary layer in slip flow, *Coll. Aero. Cranfield Rep.* no. 62, 57 pp., 2 figs., Nov. 1952.

The effect of a finite mean free path on friction and heat transfer in laminar flow becomes important at high altitudes. In this report, a first-order theory is developed. Considering only terms of order $M/R^{1/2}$, the boundary-layer equations are unchanged, but the boundary conditions at a solid surface are modified. There is a temperature jump and a finite velocity at a wall whose magnitudes are given by kinetic theory.

An approximate method of solution is employed which introduces an imaginary surface where the conventional boundary condition holds so that the conventional solution may be applied. However, the surface for the velocity and the temperature are not the same.

Typical results obtained are that, for a flat plate, there is no first-order correction to the skin friction, but the heat transfer is reduced.

However, as the author points out, the result is not valid near the leading edge where the boundary-layer equations are not valid, but where the effect of a finite mean free path should first be important.

Reviewer feels this report is a valuable contribution to a field of increasing interest. W. Squire, USA

3149. Illingworth, C. R., The laminar boundary layer associated with the retarded flow of a compressible fluid, *Aero. Res. Counc. Lond. Rep. Mem.* 2590, 23 pp., Aug. 1946, published 1953.

Two aspects of the solution of the equations governing steady gas flow in a laminar boundary layer, when the main stream velocity is nonuniform, are considered. In the first place, it is shown that the equations can be reduced to ordinary differential equations, whose solution implies the similarity of the distributions of velocity and temperature in planes perpendicular to the boundary, only in the case when the main stream velocity is uniform. In the second part, an extension of Pohlhausen's method is used to determine the point of separation of the boundary layer in an air flow in which the pressure increases with a uniform gradient. From author's summary by W. R. Sears, USA

3150. Gazley, C., Jr., Boundary-layer stability and transition in subsonic and supersonic flow, *J. aero. Sci.* 20, 1, 19–28, Jan. 1953.

See AMR 6, Rev. 587.

3151. Brown, W. B., Exact solutions of the laminar boundary layer equations for a porous plate with variable fluid properties and a pressure gradient in the main stream, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 843–852, 1952.

The three partial-differential equations governing the flow of a viscous, compressible, heat-conducting gas in a two-dimensional laminar boundary layer are transformed into two ordinary differential equations by a change of variables following Blasius, Pohlhausen, and Falkner and Skan, and with the assumptions of Mach number small, fluid properties variable with some power of absolute temperature, Euler number and wall temperature constant; the latter assumption restricts the coolant velocity to a particular variation along the wall. These simultaneous equations are solved numerically by a Picard iteration process for a range of Euler numbers (pressure gradients), temperature ratios, and various coolant air flows. Velocity and temperature distributions in the boundary layer are presented graphically, as are boundary-layer thicknesses (displacement, momentum, convection), Nusselt numbers, and wall temperatures. The computational work has been continued by Brown and Donoughe in *NACA TN* 2476 [AMR 5, Rev. 956]. J. S. Isenberg, USA

3152. Schuh, H., On asymptotic solutions for the heat transfer at varying wall temperatures in a laminar boundary layer with Hartree's velocity profiles. *J. aero. Sci.* 20, 2, 146–147, Feb. 1953.

Author refers to several papers on this subject including papers by S. Levy [AMR 6, Rev. 219] and by Chapman and Rubesin [AMR 3, Rev. 2732]. He studies asymptotic solutions for the heat transfer when the wall temperature ($T_0 = Ax^\gamma$) and the velocity outside the boundary layer ($u_1 = cx^m$) vary as power functions. The thickness of the thermal boundary layer is found to become small compared to the velocity boundary layer when either γ or the Prandtl number σ becomes large. A first asymptotic solution was obtained when $\sigma/\gamma \to 0$. A comparison between several numerical values for this solution and those of the accurate solution of Chapman and Rubesin is made. A second asymptotic solution for $\sigma \to \infty$ with no restriction on γ is expressed by a differential equation with one parameter, but no attempt was made to solve it numerically.

F. N. Frenkiel, USA

3153. Martin, J. J., Energy removal from a supersonic turbulent boundary layer, *J. aero. Sci.* 20, 2, 140–141, Feb. 1953.

Paper compares thermal power available due to aerodynamic heating of aircraft surfaces at high speeds with power required to overcome friction drag of these surfaces. Turbulent skin friction and heat-transfer coefficients calculated by Van Driest [AMR 4, Rev. 3647] are utilized. Since Van Driest's analysis shows that friction drag increases as surface temperature is reduced, author finds that heat obtained from aircraft surfaces must be utilized with fairly high efficiency ($> 40\%$ for $M = 5$) to attain a useful power greater than the power required to overcome additional friction drag due to surface cooling. W. E. Moeckel, USA

3154. Martin, J. J., Effects of angle of attack on local skin-friction coefficients at $M_0 = 4$, *J. aero. Sci.* 20, 3, 211–212, Mar. 1953.

Note treats the particular case of a flat plate at a zero angle-of-attack Reynolds number $= 1 \times 10^7$. R. E. Bower, USA

3155. Tchen, C. M., On the spectrum of energy in turbulent shear flow, *J. Res. nat. Bur. Stands.* 50, 1, 51–62, Jan. 1953.

Paper is an attempt to derive the spectrum of turbulent energy for large wave numbers, in the equilibrium range, under conditions of strong mean shearing motion, the ultimate new result being that the energy spectral density is proportional to the reciprocal of the wave-number magnitude in the inertial subrange. The argument assumes that the only effect of the mean shearing motion is to contribute to local mean-square vorticity, that the mean-square vorticity associated with eddies above a certain size is dominated by the product of mean vorticity and root-mean-square turbulent vorticity, and that the Weizsacker-Heisenberg form of the expression for transfer of energy across the spectrum is valid.

The general physical picture assumed by the author was not clear to this reviewer, and it is difficult to assess the probable validity of the result. Nor is the author's comparison of his results with measurements at N.B.S. without ambiguity; *some* agreement between an experimental curve and a power law can always be achieved on a logarithmic plot if the experimental curve changes its slope continuously.

G. K. Batchelor, England

3156. **Pai, S. I., On turbulent flow between parallel plates,** *J. Appl. Mech.* **20,** 1, 109–114, Mar. 1953.

A semi-empirical formula for the mean velocity is given and checked with experimental data in the whole region including the viscous layer. It is an approximate formula again and not a rigorous one since, e.g., the corresponding turbulent shearing stress remains finite in the laminar sublayer except at the wall itself. K. Wieghardt, Germany

3157. **Uberoi, M. S., Quadruple velocity correlations and pressure fluctuations in isotropic turbulence,** *J. aero. Sci.* **20,** 3, 197–204, Mar. 1953.

Diffusion and sound generation in turbulent flow are related to the fluctuations in the static pressure. In incompressible homogeneous fields these fluctuations are predictable from the small difference between certain fourth-order correlations of the velocity. This work assumes an explicit relation between fourth-order and second-order velocity correlations and supports this assumption with experimental data. However, the relation between rms velocity and rms pressure fluctuation predicted on the basis of this assumption agrees rather poorly with the observations. W. V. R. Malkus, USA

3158. **Liepmann, H. W., Parameters for use in buffeting flight tests,** *Douglas Aircr. Co. Rep.* SM-14631, 8 pp., Jan. 1953.

An extension of the application of statistical concepts to the buffeting problem [*J. aero. Sci.* **19,** 12, 793–800, Dec. 1952] for special conditions shows the possible significance of aerodynamic and structural damping on the root-mean-square buffeting stress. For aerodynamic surfaces with one natural frequency ω_0 and low damping, the analysis indicates the root-mean-square stress to be proportional to the damping and some function of the reduced frequency $c\omega_0/U$. If the damping is primarily structural, the root-mean-square stress varies directly with density and as the 3/2 power of the forward speed. For aerodynamic damping, the stress varies as the half power of air density and directly with the forward speed. For experimental work, the concepts given can be useful in interpreting the significant features of an aerodynamic surface in a turbulent wake. P. Donely, USA

Aerodynamics of Flight; Wind Forces

(See also Revs. 3136, 3137, 3139, 3143)

3159. **Dommasch, D. O., Elements of propeller and helicopter aerodynamics,** New York, Toronto, London; Pitman Publ. Corp., 1953, xii + 178 pp.

The book is intended primarily for use as a textbook in undergraduate courses of aeronautical engineering.

An introductory chapter first presents some of the basic principles and definitions of aerodynamics and then gives a representation of incompressible and compressible axial momentum theory of propellers. The next chapter deals with blade element theory. The results of Goldstein's theory are incorporated. A few remarks about the effects of compressibility on propeller characteristics are also included. The third chapter is devoted to a discussion of propeller characteristics and to the use of design charts. Some attention is given to dual rotating and ducted propellers. Finally, the behavior of a propeller in pitch and yaw is studied.

The second half of the book deals with the aerodynamics of helicopters. Chap. 4 discusses vertical flight, hovering, and autorotation. Chap. 5 presents the theory of forward flight. Calculation of the flapping motion of the blades, however, is omitted. The final chapter contains a discussion of static stability in hovering and forward flight.

No mathematics beyond the elements of calculus is used. The book can be recommended as a first introduction to the subject. H. Parkus, USA

3160. **Stokes, F. H., and Matthews, J. T., Theoretical investigation of the longitudinal response characteristics of a swept-wing fighter airplane having a pitch-attitude control system,** *NACA TN* 2882, 41 pp., Jan. 1953.

Authors have theoretically investigated an automatic control system with pitch-attitude feedback alone and pitch-attitude plus pitch-rate feedback. Choice of optimum gains as affected by variations in Mach number and altitude is discussed. Based on the characteristics of the frequency and transient responses in pitch attitude, authors conclude that a gain computer would be required for the autopilot-airplane combination to perform well for all flight conditions. The pitch-attitude autopilot is not suitable for precise flight-path control. Conclusions are somewhat limited by the assumption of a completely linear system. W. F. Milliken, Jr., USA

3161. **Buhrman, J., and Kalkman, C. M., The static longitudinal stability and control of an aeroplane as affected by the compressibility of the air,** *Nat. LuchtLab. Amsterdam Rep.* V.1625, 10 pp., 1951.

The three-dimensional Glauert-Prandtl rule is used to derive formulas for the estimation of the static longitudinal stability and control characteristics of airplanes with unswept wings at subcritical speeds. The results of some numerical applications of these formulas are presented.

Both the stick-fixed and stick-free static margins are shown to decrease in general with increasing Mach number, and occasionally, to assume large negative values. The maneuver margin tends to decrease slightly with Mach number in most cases. The rather large negative static margins caused by subcritical compressibility effects are stated to be generally acceptable, provided the maneuver margin remains positive. However, it should be pointed out that prolonged flight under these conditions may put undue strains on the pilot. F. W. Diederich, USA

3162. **Jacobs, W., and Truckenbrodt, E., The induced side wash of airplanes** (in German), *Ing.-Arch.* **21,** 1, 1–22, 1953.

The side wash due to sideslip is explained as follows: The flow about a yawed fuselage produces an antisymmetrical contribution to the wing spanwise lift distribution, of opposite sign for high- and low-wing arrangements and nominally zero for mid-wing. The trailing vortexes due to this contribution are strong near the fuselage and produce the side wash at the tail. Evaluation of these effects is important for the estimation of directional stability and similar matters. A calculation is made, using plausible approximations, and curves of side-wash angle are presented for several geometrical arrangements. In these curves, no account is taken of the (first-order) displacement of the trailing vortexes that would actually occur.

The results of a series of wind-tunnel observations are then presented. The tests bear out the predictions of the theory, at least qualitatively and to some extent quantitatively. The side-wash effect on directional stability is stabilizing for low-wing, destabilizing for high-wing, and about neutral for mid-wing monoplanes. W. R. Sears, USA

3163. **Donegan, J. J., Matrix methods for determining the longitudinal-stability derivatives of an airplane from transient flight data,** *NACA TN* 2902, 65 pp., Mar. 1953.

Problem considered is determination of dynamic longitudinal stability of an airplane from flight test measurements. Two de-

grees of freedom, vertical motion, and pitch are assumed, as well as the usual linearized form of the equations of motion. Three matrix procedures are developed involving measurements of certain quantities related to the time history in flight, as well as further simplifying assumptions. The first method requires four measurements and no additional assumptions; the second method involves three measurements and one supplementary assumption; the third procedure involves two measurements and two assumptions. Results obtained by the three methods based on actual flight-test data are compared. In all cases, high accuracy of measurement is essential. The first method is the most satisfactory, but the second is frequently a good alternate, considering the amount of work required in computation.

M. J. Thompson, USA

3164. Schy, A. A., and Gates, O. B., Jr., A theoretical method of analyzing the effects of yaw-damper dynamics on the stability of an aircraft equipped with a second-order yaw damper, *NACA TN* 2857, 72 pp., Dec. 1952.

Yaw dampers have been used recently to improve the damping of the Dutch roll oscillation of high-speed aircraft. Report presents a detailed theoretical analysis of the relations between the characteristics of a second-order autopilot yaw damper and the gain required for a given Dutch roll damping. Application of these analytical relations to a typical aircraft is shown. Results indicate that a particular choice of damping and natural frequency of the autopilot will allow a lower gain to achieve the same Dutch roll damping as an ideal (zero phase lag) autopilot. This lowered gain will decrease the undesirable rudder forces in steady turns. Reviewer believes that the simplified method presented should prove valuable to engineers concerned with yaw-damper design.

F. W. Heilenday, USA

3165. Kraft, C. C., Jr., Goranson, R. F., and Reeder, J. P., Measurements of flying qualities of an F-47D-30 airplane to determine longitudinal stability and control and stalling characteristics, *NACA TN* 2899, 75 pp., Feb. 1953.

Paper, originally issued in 1948, contains flight test data on longitudinal stability and control, and stalling characteristics of F-47 fighter airplane. Results were compared to Air Force stability and control requirements. H. Luskin, USA

3166. Hirsch, R., Contribution to the problem of automatically controlled airplane landing (in French), *Rech. aéro.* no. 30, 19–24, Nov.-Dec. 1952.

Paper is concerned with an analysis of the lateral dynamic stability characteristics of an airplane during a controlled flight approach to a landing. The equations of motion are presented in the usual linearized form, considering yawing, sideslip, and rolling motions. The case of an airplane in free flight is first considered, then that of a controlled flight. In the latter case, two situations are discussed: (1) Automatic pilotage with maintenance of course; and (2) control along a specified trajectory. There is a brief discussion of related wind-tunnel and flight tests.

M. J. Thompson, USA

3167. Nicolaides, J. D., and Bolz, R. E., On the pure rolling motion of winged and/or finned missiles in varying supersonic flight, *J. aero. Sci.* 20, 3, 160–168, Mar. 1953.

Consideration is given to effect of refinement of the equation of motion used to evaluate the three roll-characteristics C_{l_p}, C_{l_δ}, and $pb/2V$ from observations of roll position vs. longitudinal position in flight. The simpler equation, which neglects the variation of Mach number during a flight, is found entirely adequate for use with a firing range such as here considered. Test results for 8% thick rectangular fins are compared with previous results for 16% thick fins, and with results of linearized theory; results seem satisfactorily consistent. It is demonstrated that presentation of C_{l_p} and C_{l_δ} vs. Mach number can be simplified by plotting the *reciprocals* of these coefficients instead of the coefficients themselves. Test results are also reported for the effect of adding tail fins to a cone-cylinder body which already carries a set of wings. R. H. Norris, USA

3168. Bolie, V. W., Powerless glides along zero curvature flight paths, *J. Franklin Inst.*, 255, 2, 127–132, Feb. 1953.

Formulas are obtained for speed and air distance at any instant during a straight glide on the assumption that both lift and drag coefficients are linear functions of incidence. Although reasonable agreement with experiment is obtained for example in text, reviewer feels that basic physical principles, expressed by the standard formula $C_D = a + bC_L^2$, have been sacrificed to introduce mathematical simplicity. In general, it will be necessary to accept that the drag coefficient is a function of incidence squared, and then proceed by more classical methods for step-by-step integration. S. Kirkby, England

3169. Margolis, K., Sherman, W. L., and Hannah, Margery E., Theoretical calculation of the pressure distribution, span loading, and rolling moment due to sideslip at supersonic speeds for thin sweptback tapered wings with supersonic trailing edges and wing tips parallel to the axis of wing symmetry, *NACA TN* 2898, 70 pp., Feb. 1953.

Theoretical calculations are made for the title problem, based on linearized supersonic-flow theories of Evvard and conical flow. Numerical results are given in form of graphs of spanwise loadings and variation of the stability derivative of rolling moment due to sideslip with Mach number for a range of aspect ratio, taper ratio, and leading-edge sweepback. H. G. Lew, USA

3170. Heaslet, M. A., and Lomax, H., The calculation of pressure on slender airplanes in subsonic and supersonic flow, *NACA TN* 2900, 25 pp., Mar. 1953.

Under the assumption that a wing, body, or wing-body combination is slender or flying at near sonic velocity, expressions are given which permit calculation of pressure in the immediate vicinity of the configuration. The disturbance field, in both subsonic and supersonic flight, is shown to consist of two-dimensional disturbance fields extending laterally and a longitudinal field that depends on the streamwise growth of cross-sectional area. A discussion is also given of couplings, between lifting and thickness effects, that necessarily arise as a result of the quadratic dependence of pressure on the induced velocity components.

From authors' summary by T. R. Gullstrand, Sweden

3171. Walker, K., Jr., Examples of drag reduction for rectangular wings, *Douglas Aircr. Co. Rep.* SM-14446, 49 pp., Jan. 1953.

Author applies Graham's method for drag reduction [AMR 6 Rev. 1974] to examine the effect of twist, camber, and thickness distribution on the induced drag of thin wings of rectangular planform at small angle of attack in nonviscous supersonic flow. Twist is found to be ineffective in reducing drag, whereas camber is most effective in the tip region. Drag reductions are largest for wings of low reduced aspect ratio $AR(M^2-1)^{1/2}$, with theoretical reductions of as much as 36% for reduced aspect ratio 1.2. Smaller improvements are obtained when adjusting thickness dis-

tribution, again being most favorable for low reduced aspect ratio. P. Eisenberg, USA

3172. **Castles, W., Jr., and De Leeuw, J. H., The normal component of the induced velocity in the vicinity of a lifting rotor and some examples of its application,** *NACA TN* 2912, 38 pp., Mar. 1953.

Based on the assumption that vortexes are shed only from the blade tips, a method is presented for the evaluation of the normal component of the induced velocity at points in the flow field of a lifting rotor. Tables and graphs of the relative magnitudes of the normal component of the induced velocity are given for points in the longitudinal plane of symmetry and on the lateral rotor axis. A method is also presented for utilizing the tables and graphs to determine the interference-induced velocities arising from the second rotor of a tandem or side-by-side rotor helicopter and the induced flow angle at a horizontal tail plane.

It is shown, in part, that the interference-induced velocity at the rear rotor of a tandem-rotor helicopter in high speed, due to the thrust of the front rotor, is of the same order of magnitude and of the same sign as the self-induced velocity. L. Goland, USA

3173. **Teofilato, P., Maximum efficiency windmills** (in Italian), *Monogr. sci. Aero.* no. 13, 12 pp., Jan. 1953.

Problem of maximum efficiency of windmills without the hypothesis of the cylindrical jet is studied. General equations of continuity, energy, and momentum are applied as usual. From this mathematical treatment it is obtained that both the circulation and the axial increment of the velocity at downstream infinity are nearly constant along the radius. On the contrary, the axial increment at the windmill varies with the radius.

In this way the maximum calculated efficiency is greater than that obtained by Glauert with the hypothesis of the cylindrical jet. From author's summary by A. Balloffet, Argentina

3174. **Callaghan, E. E., and Serafini, J. S., A method for rapid determination of the icing limit of a body in terms of the stream conditions,** *NACA TN* 2914, 33 pp., Mar. 1953.

An energy balance is made for a point on an insulated nonconducting airfoil in which the aerodynamic (frictional) heating is equated to the energy losses by evaporation and heat convection. The parameters in the energy balance are the local pressure coefficients over the wing and the stream conditions. Charts are presented which permit a particular wing to be analyzed (if pressure coefficients are known) to determine the flight speed at which the surface is maintained wet and ice free at 32 F.

The reviewer believes results are incomplete, since charts for high-speed aircraft may indicate a danger from icing when, in fact, the danger does not exist, since small amounts of water in the atmosphere may be evaporated. Charts do not investigate mass balance between catch rate and evaporation.

Of the ten references, nine are to NACA works with no mention of the summary work of Messinger which covers the same ground in more thorough fashion and upon whose efforts the present report appears to be based. M. Tribus, USA

Aeroelasticity (Flutter, Divergence, etc.)

3175. **Merbt, H., and Landahl, M., Aerodynamic forces on oscillating low aspect ratio wings in compressible flow,** *Roy. Inst. Technol., Div. Aero., KTH Aero. TN* 30, 20 pp., 1953.

The linearized equation for unsteady flow reduces to the two-dimensional wave equation for slender wings. By using elliptical coordinates, author shows that the wave equation can be reduced to two equations of the same form as Mathieu's equation. Results are applied to an oscillating delta wing for various aspect ratios and Mach numbers, the force and moment coefficients being determined for both vertical translation and pitching. The range of validity of these solutions is examined and it is shown that the solutions are more restricted than the corresponding stationary solutions. For sufficiently small values of the aspect ratio, the theory is valid for all frequencies. At the sonic speed, the theory may be valid for wings of arbitrary aspect ratio for both very low and very high frequencies.

A. W. Babister, Scotland

3176. **van de Vooren, A. I., Aerodynamic coefficients of an oscillating airfoil with control surface in two-dimensional subsonic flow,** *J. aero. Sci.* 20, 3, 153–159, Mar. 1953.

Tables are presented for the moments and forces arising from two-dimensional subsonic compressible flow on a flap and on a wing arising from flap oscillations. A range of frequency parameters ω is covered up to approximately 1.0 for Mach numbers ranging from 0.35 to 0.8 for three flap chord ratios. These tables are meant to supplement an earlier paper by Timman, van de Vooren, and Greidanus, which gave tabulated values for the lift and moment derivatives in two-dimensional subsonic compressible flow on a wing alone. The author maintains that all tabulated values should be accurate to one unit in the last decimal place supplied. A short discussion is presented which describes the method for handling the singularity at the leading edge of the control surface.

The tabulated results obtained by the exact method used by this author in the present paper and in the earlier paper in which he collaborated have been subjected to criticism by Fettis and Jordan in the Readers Forum of the *IAS Journal*, since these results do not check the tabulated results obtained by the approximate Dietze method. It is hoped that this issue will be clarified soon, since the tables presented are by far the most complete yet available to the flutter analyst. J. E. Stevens, USA

3177. **Pengelley, C. D., and Benun, D., Aeroelastic studies on a high-performance, swept-wing airplane,** Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 903–906, 1952.

Wind-tunnel tests on a complete aeroelastic model of an airplane with swept wing and tail surfaces are compared with the theoretical analysis set forth by authors in another paper. Good agreements are obtained in longitudinal stability and control, but appreciable quantitative discrepancies occur for lateral conditions. Fuselage flexibility is also discussed. S. I. Pai, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 3159, 3201, 3206, 3208, 3245)

3178. **Hanson, M. P., Meyer, A. J., Jr., and Manson, S. S., A method of evaluating loose-blade mounting as a means of suppressing turbine and compressor blade vibration,** *Proc. Soc. exp. Stress Anal.* 10, 2, 103–116, 1952.

Authors discuss effectiveness of loose-blade mounting as a means of damping blade vibration. Measurements of damping were made at a blade mounted on a rotating disk. Difference of logarithmic damping between loose-blade and fixed-blade is ascribed to damping effect of loose-mounting. In each case, damping was calculated from the breadth of the frequency response curve, i.e., the plot of vibration amplitude against frequency at constant excitation. Loose-blade damping is found to

be effective only at low rotative speeds. Use of solid lubricants extends the effectiveness to higher speeds.

Reviewer feels the term "loose" rather vague here. It needs more exact quantitative description.

L. S. Dzung, Switzerland

3179. Allen, H. W., Kofskey, M. G., and Chamness, R. E., Experimental investigation of loss in an annular cascade of turbine-nozzle blades of free vortex design, *NACA TN* 2871, 33 pp., Jan. 1953.

Measurements of total pressure distribution and of flow direction were made at exit of cascade having discharge angle near 65° and with exit hub Mach numbers of 0.94 and 1.46. High loss regions were found at junction of blade wakes and shroud boundary layers. Substantial secondary flows occurred. Overall integrated blade efficiencies were 0.99 (at $M = 0.94$) and 0.98 (at $M = 1.46$); however, these are not considered a good measure of blade performance because of adverse effects of secondary flow, wake losses, and flow-angle deviations on behavior of following row. A. H. Shapiro, USA

3180. Davis, D. D., Jr., Stevens, G. L., Jr., Moore, D., and Stokes, G. M., Theoretical and measured attenuation of mufflers at room temperature without flow, with comments on engine-exhaust muffler design, *NACA TN* 2893, 111 pp., Feb. 1953.

Authors present a thorough quantitative analysis of simple chamber, multiple chamber, and combination mufflers. Equations for attenuation and cutoff frequency are derived in terms of the significant physical parameters. Design curves for the mufflers are developed, and the effect of expansion ratio, length, and shape is discussed. Comparison is made between theoretical predictions and experimental results, and close agreement is found. The material contained should be of interest both to those interested in theoretical aspects of sound attenuation and to designers of sound-attenuation equipment.

J. F. Manildi, USA

3181. Sears, W. R., On asymmetric flow in an axial-flow compressor stage, *J. appl. Mech.* **20,** 1, 57–62, Mar. 1953.

Paper presents simplified theoretical analysis showing that subject phenomenon may occur if a large time lag exists in the development of airfoil lift (as is possible when the blades are stalled). Further study is under way of effects caused by nonlinear airfoil characteristics. B. Smilg, USA

3182. A rotary compressor with inbuilt compression, *Engineer, Lond.* **195,** 5062, 170–171, Jan. 30, 1953.

The development of a new design of a positive variable displacement compressor by the British Internal Combustion Engine Research Association (BICERA), a rotary machine with inbuilt compression, is described. Comparisons are made with the Roots pressure-charging system; design considerations are reviewed; and a specification for a prototype compressor is given together with test results.

From author's summary by H. W. Iversen, USA

3183. Eck, B., Fans. Design and operation of centrifugal and propeller blowers [Ventilatoren. Entwurf und Betrieb der Schleuder- und Schraubengebläse] 2nd ed., Berlin, Springer-Verlag, 1952, xi + 304 pp. DM 36.

This second and much enlarged edition (first edition published in 1937) contains extensive information on the design of fans, especially of the radial-flow type. Book is divided into three main parts. Part A on radial-flow fans takes up more than half of the book. Starting with the usual infinite-vane theory, the book quickly leads to many practical topics, such as the influence of finite number of blades, similarity relations, properties of different types, estimation of losses, vane shapes, runner shapes, performance, guide vanes, spiral casing, and special designs. Part B on axial-flow fans covers the usual lattice theory, airfoil theory, friction and loss in runner and in diffuser, diffuser shapes, performance, meridionally accelerated (mixed-flow) fans, etc. Part C describes construction details of many existing designs, illustrated with some 70 photographs and cross-section drawings. Near the end of the book there are also two short chapters, one on stress calculation and the other on testing.

This book is written primarily for the designer who does not care for highly mathematical theory. Essence of theory is expressed in simple forceful terms. H. Yeh, USA

Flow and Flight Test Techniques

(See also Revs. 3158, 3165, 3167, 3226)

3184. Liepmann, H. W., and Dhawan, S., Direct measurements of local skin friction in low-speed and high-speed flow, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 869–874, 1952.

Paper discusses development and application of a device to measure directly local skin friction on a flat plate. Device consists, essentially, of a movable element mounted flush with surface of flat plate. Shear forces exerted by fluid flow cause very small longitudinal translation of element. This deflection, which is proportional to force exerted on element, is measured by a reluctance pickup.

Results and conclusions are presented for actual wind-tunnel investigation utilizing above device at subsonic and supersonic speeds. The data show good agreement with the von Kármán and Blasius theory, and indicate that the method is feasible but that further experiments are necessary to establish reliability at supersonic speeds. H. L. Bloom, USA

3185. Preston, J. H., Gregory, N., and Kimber, K. W., Transition and drag measurements on the Boulton Paul sample of laminar-flow wing construction, *Aero. Res. Counc. Lond. Rep. Mem.* 2499, 23 pp., Nov. 1946, published 1952.

Paper describes tests correlating the position of boundary-layer transition (using chemical evaporation and china-clay techniques) with the drag of a laminar-flow wing (NACA 65.3.018 section). Tests of the same model were conducted in two wind tunnels with good agreement. Agreement with theoretical predictions is also good. The effects of skin waviness and other surface imperfections are described. M. S. Macovsky, USA

3186. Michel, R., Determination of the transition point and calculation of the friction drag of airfoils in incompressible flow (in French), *ONERA Publ.* no. 58, 53 pp., 3 tables, 16 plates, 1952.

Paper presents an interesting review of common methods of calculating and testing transition point and friction coefficient of airfoils in incompressible media. Navier-Stokes equations in Prandtl's form are assumed as fundamental. In part I, author reviews known methods of calculating boundary-layer thickness, displacement thickness, friction coefficient, etc., in laminar and turbulent layers, as well as separation and transition points. In part II, author reviews various experimental methods of transition—visualization, acoustic, heat, Jones's criterion, etc. He emphasizes the necessity of replacing the several current definitions of transition point with a single universally accepted one. He exploits experimental results obtained in ONERA wind tun-

nels for calculation of transition point (part III) and friction drag of symmetric airfoil section (part IV). In part V, he discusses the role of universal curvatures, allowing one to calculate transition and their application to calculation of friction coefficient of total drag (part VI). Author emphasizes the importance of Reynolds number based on the momentum thickness of the boundary layer in evaluating transition point by the procedure proposed by himself. Numerous tables and diagrams close the paper.

M. Z. Krzywoblocki, USA

3187. **Blue, R. E., and Low, G. M., Factors affecting laminar boundary layer measurements in a supersonic stream,** *NACA TN* 2891, 49 pp., Feb. 1953.

Report is an excellent experimental study of the boundary layer on two flat plate models at free-stream Mach numbers of 2 and 3. Purpose was to explain the discrepancy between theoretical and experimental friction-drag coefficients of a flat plate at supersonic speeds. The experimental data were obtained by total-pressure surveys of the boundary layer and gave drag coefficients that were 10 to 100% higher than the theoretical values calculated with the compressible laminar boundary-layer theory. Possible causes of this discrepancy are: Effects of the total-pressure probe, heat transfer through the leading edge, curvature of the leading edge, possible early transition to turbulent flow, etc. Authors examined all these factors and show clearly that only the effect of the total-pressure probe is significant. A range of probe trip heights from 0.003 to 0.016 in. under identical conditions gave different values of friction-drag coefficients. Extrapolation to zero height leads to agreement with the theoretical values.

K. Pohlhausen, USA

3188. **Donaldson, C. du P., and Sabol, A. P., Experiments on aerodynamic phenomena at Mach numbers in the range from 10 to 20,** Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 757–762, 1952.

Using a small ballistics range, projectiles were fired from a 30-caliber rifle through krypton and xenon. Tests of spheres and cones at Mach numbers between 10 and 20 indicated ionization of the atoms past which the bodies traveled. At Mach numbers above 10 there appear to be certain flow phenomena which are not present at much lower Mach numbers. R. C. Binder, USA

3189. **Higgs, H. J., Some investigations into the design of wind tunnels with gas turbine jet engine drives,** *Aero. Res. Counc. Lond. curr. Pap.* 107, 60 pp., 28 figs., 1953.

The purpose is to collate some of the information on these tunnels, to examine the various theories in order to obtain a basis for future designs, and to examine the possibilities of extended use of the system to give a supersonic tunnel. The methods of drive investigated are (a) to use the jet as the driving jet in an induction pump, (b) to couple the tunnel directly to the engine air intake and to use the engine as a pump, and (c) to combine (a) and (b) so that the engine is contained in the tunnel with flow around the engine as well as through it. Some of the topics discussed are possible configurations of wind tunnels, basic design considerations, subsonic and supersonic tunnels, humidity control, determination of induction-pump characteristics, friction loss in mixing chamber, matching of tunnel and induction-pump characteristics, determination of suction-pump characteristics, matching of tunnel and suction-pump characteristics, and jet engine drive by suction and pumping in paralliel. The necessary equations are developed step by step and the assumptions made are clearly stated. Numerous charts contain both necessary design information and results of computations made by equations derived in the paper. To illustrate the use of the methods and charts, the determination of the essential parameters is worked out in detail for four wind-tunnel configurations.

[For additional information, see AMR **4**, Rev. 3673.]

N. Tetervin, USA

3190. **Bernstein, H., Effects of parallel-jet mixing on downstream Mach number and stagnation pressure with application to engine testing in supersonic tunnels,** *NACA TN* 2918, 26 pp., Mar. 1953.

Two parallel nonviscous streams are mixed at constant area resulting in a final uniform flow. The properties of this downstream flow are determined by a one-dimensional analysis. The method accounts for compressibility and applies to subsonic and supersonic streams. In general, two solutions are obtained; a supersonic final stream and a subsonic flow corresponding to the condition after a normal shock.

A calculation is carried out for a heated jet mixing with wind-tunnel air. The final flow conditions indicate the performance requirements for operation of the wind tunnel. In this calculation, the specific heats are assumed constant, but the author shows that only a small error is thereby introduced.

The ordinate scale in Fig. 1 is in error and should be shifted downward by 0.01. L. H. Schindel, USA

3191. **Cooper, M., and Sevier, J. R., Jr., Estimated power reduction by water injection in a nonreturn supersonic wind tunnel,** *NACA TN* 2856, 19 pp., Jan. 1953.

Paper presents a simplified theoretical analysis estimating operating-efficiency improvement of a nonreturn supersonic wind tunnel by water injection behind the test section. Assuming one-dimensional flow, perfect gas and isentropic-flow conditions on each side of the water injection in the diffusor subsonic-flow region, the operating pressure ratio and tunnel-power reduction to be realized by cooling are calculated.

For a typical wind tunnel operating in the lower supersonic Mach number range, the gains in power saving are shown to be relatively large for the amounts of water required and are likely to outweigh the losses associated with the disturbed flow conditions in the diffusor and the power to provide the necessary water.

Yap Kie Jan, Holland

3192. **Syvertson, C. A., and Savin, R. C., The design of variable Mach number asymmetric supersonic nozzles by two procedures employing inclined and curved sonic lines,** *NACA TN* 2922, 35 pp., Mar. 1953.

Formerly published as *NACA RM* A51A19. A noniterative design procedure, based on the method of characteristics, provides a wind-tunnel nozzle in which the Mach number may be varied by sliding the lower block with respect to a fixed upper one. The test section does not move. The procedure produces theoretically uniform flow at two arbitrarily selected Mach numbers. The flow at off-design Mach numbers is made satisfactorily uniform by designing the blocks with continuous curvature. A similar procedure, but requiring iteration, is suggested for high Mach numbers (above 3).

Coordinates are presented (not including boundary layer) for a nozzle designed for uniform flow at Mach numbers 1.5 and 2.6. Calibration data on this nozzle are presented at Mach numbers 1.49, 1.89, 2.28, and 2.62.

Reference 1 should read *TN* 2919. L. H. Schindel, USA

3193. **Beastall, D., and Winyard, A., A frost point hygrometer for supersonic wind tunnels,** *Aero. Res. Counc. Lond. curr. Pap.* 106, 4 pp., 4 figs., July 1951, published 1953.

This note describes a frost point hygrometer suitable for meas-

uring the water vapor content of the air in supersonic wind tunnels at any stagnation pressure within their present range of operation. It uses CO_2 as a coolant and is economical in construction and operation. From authors' summary

3194. Li, Y. T., and Lee, S. Y., A fast-response true-mass-rate flowmeter, *Trans. ASME* **75,** 5, 835–841, July 1953.

Paper surveys available flowmeters and describes design of, and experiments with true-mass-rate flowmeters. These use the principle of giving the flow a Coriolis acceleration and measuring the torque produced with strain gages of special design. Two models are described. They had maximum flow ratings of 5 to 10 gpm, rotor diameters of 5 in., and operated at 1800 rpm. In one model, the flow-sensing element was semi-Y-shaped; in the other, it was a T-tube in a heavy housing. The experimental tests gave excellent agreement with theoretically predicted characteristics. The new type of mass-rate flowmeter has a long range, a linear scale, and a very fast response, but it requires a constant-speed motor and two rotary fluid seals.

N. H. Johannesen, England

3195. Coss, B. A., Daykin, D. R., Jaffe, L., and Sharp, E. M., A digital multiple pressure recorder, *NACA TN* 2880, 24 pp., Jan. 1953.

A machine is described which will automatically measure and record 100 pressures in a range from 5 to 65 in. of mercury, in approximately $2^1/_2$ min, to an accuracy of 0.1 in. of mercury. The method used is to compare the unknown pressures with a scanning pressure whose value at any instant is known in digitalized form. Sensitive diaphragms indicate balance between the unknown and the scanning pressures. All unknown pressures are compared with the scanning pressure simultaneously, and the information is stored temporarily within the machine. During read-out, the information is properly sequenced, identified, coded, and punched into paper tape, which is the actual permanent record of the output of the machine, although typewritten tabulated data may also be produced. From authors' summary

3196. Pressey, D. C., Temperature-stable, capacitance pressure gauges, *J. sci. Instrum.* **30,** 1, 20–24, Jan. 1953.

A brief theoretical treatment is given of the effect of temperature changes on capacitance pressure gages. Its use in the design of a sensitive differential pressure gage of range 0–10 mm of water, having a temperature coefficient of capacitance of only -100 ppm/° C, is illustrated. From author's summary

3197. Gross, N., and Lane, P. H. R., An accurate wire resistance method for the measurement of pulsating pressures, *J. sci. Instrum.* **30,** 1, 1–2, Jan. 1953.

A method of measuring pulsating pressures up to 6000 psi by a wire-resistance method is described. The pressure-sensitive element consists of a cylindrical steel tube with a wire winding, incorporated as an arm in a bridge circuit, the output from which is fed without amplification to a mirror galvanometer with a natural frequency of 10 cps. The records show that a high degree of linearity and stability can be achieved with this combination.

From authors' summary

3198. Kovásznay, L. S. G., Development of turbulence-measuring equipment, *NACA TN* 2839, 86 pp., Jan. 1953.

Hot-wire turbulence-measuring equipment has been developed to meet the more stringent requirements involved in the measurement of fluctuations in flow parameters at supersonic velocities. The higher mean speed necessitates the resolution of higher frequency components than at low speed, and the relatively low turbulence level present at supersonic speed makes necessary an improved noise level for the equipment. The equipment covers the frequency range from 2 to about 70,000 cps. Constant-current operation is employed. Compensation for hot-wire lag is adjusted manually, using square-wave testing to indicate proper setting. These and other features make the equipment adaptable to all-purpose turbulence work with improved utility and accuracy over that of older types of equipment. Sample measurements are given to demonstrate the performance. Complete descriptions and wiring diagrams are given.

From author's summary by C. F. Bonilla, USA

3199. Engel, F. C., Fuel spray examination methods, ASME Ann. Meet., New York, Dec. 1952. Paper no. 52—A-139, 7 pp., 15 figs.

Methods of examination of liquid sprays are discussed. Samplers for the determination of spray angle, spray uniformity and symmetry are described. A photographic arrangement for obtaining spray-droplet size distribution is presented and sample pictures are shown. From author's summary

3200. Fletcher, G. L., Tank tests on a jet-propelled boat-seaplane fighter (Saunders-Roe E6/44), *Aero. Res. Counc. Lond. Rep. Mem.* 2718, 31 pp., Jan. 1946, published 1952.

Report covers hydrodynamic tank-testing of the world's first jet-propelled seaplane. Two dynamically similar models are used of differing scales. This allows a study to be made of the influence of scale effect in addition to porpoising stability, resistance, and seaworthiness. Many unsatisfactory hydrodynamic characteristics are corrected by progressive analysis and modification. A satisfactory design is evolved with adequate stability and seaworthiness for sheltered water operation. This report is an excellent example of modern high-speed hydrodynamic testing procedure and analysis. E. G. Stout, USA

3201. Jasper, N. H., and Rupp, L. A., An experimental and theoretical investigation of propeller shaft failures, *Soc. nav. Arch. mar. Engrs.*, 52 pp., Nov. 1952.

Thermodynamics

(See also Rev. 3145)

3202. Paterson, S., Propagation of a boundary of fusion, *Proc. Glasgow math. Assoc.* **1,** 42–47, 1952.

The problem considered is that of the propagation of a boundary of fusion in the two-phase case. A volume of material is divided into two regions 1 and 2, each of density ρ, by a moving surface S. On S a change of phase occurs at a definite temperature, zero. If θ_i, k_i, K_i $(i = 1, 2)$ are the temperature, thermal conductivity, and diffusivity of phase i, then the following equations must hold

$$\frac{\partial \theta_i}{\partial t} = K_i \nabla^2 \theta_i \quad (i = 1, 2) \tag{1}$$

with the boundary conditions on S

$$\theta_1(x, y, z, t) = \theta_2(x, y, z, t) = 0 \tag{2}$$

$$k_1 |\text{grad } \theta_1| - k_2 |\text{grad } \theta_2| = \pm \frac{L\rho}{|\text{grad } \theta_1|} \frac{\partial \theta_1}{\partial t} = \pm \frac{L\rho}{|\text{grad } \theta_2|} \frac{\partial \theta_2}{\partial t} \tag{3}$$

The condition for a solution is that θ_i separately satisfy

$$\nabla^* \frac{\text{grad}^2\,\theta}{\partial\theta/\partial t} = \text{const, when } \theta = 0 \qquad [4]$$

In the one-dimensional cases of linear, axially symmetric, or spherically symmetric flow, Equations [1] reduce to

$$\frac{\partial\theta_i}{\partial t} = K_i\left[\frac{\partial^2\theta_i}{\partial r^2} + \frac{n}{r}\frac{\partial\theta_i}{\partial r}\right] \quad (i = 1, 2) \qquad [5]$$

with $n = 0, 1, 2$ in the three cases, respectively. Further, condition [4] reduces to

$$\left(\frac{\partial\theta}{\partial r}\right)^2 \Big/ \frac{\partial\theta}{\partial t} = \text{const, at } \theta = 0 \qquad [6]$$

Using the known solutions for these three cases, the author exhibits solutions for the propagation of a boundary of fusion from a line source of heat in an infinite fusible solid and from a point source. An ice-water system is given as an example of the first problem, and a temperature profile chart is constructed. It is pointed out that a boundary of vaporization should follow the boundary of fusion, and an indication of the solution in this case is given.

Courtesy of Mathematical Reviews C. G. Maple, USA

3203. **Rushbrooke, G. S., and Scoins, H. I., On the theory of fluids,** *Proc. roy. Soc. Lond.* (*A*) **216,** 1125, 203–218, Jan. 1953.

The theory of fluids based on the radial distribution function $g(r)$ is related to the direct correlation function $f(r)$ of scattering theory.

The simplest form of the Born-Green theory is shown to be intimately related to the first approximation to $f(r)$. The second approximation to $f(r)$ is considered in detail, and shown to effect a considerable improvement on earlier theories. It leads to a critical point at which $p/\rho kT = 1/3$.

Fournet's application of the Born-Green theory to x-ray scattering by fluids is shown to be correct only to the first power in the density. The terms in ρ and ρ^2 are considered in detail for the case of hard spheres. From authors' summary

3204. **Curtiss, C. F., The kinetic theory of dense gases,** *Univ. Wisc., Nav. Res. Lab.* OOR-3, 46 pp., Jan. 1953.

An approximate kinetic theory of dense gases is developed. The theory is a modification of the Enskog theory of a dense gas made up of rigid spherical molecules and takes into account the effect of "collisional transfer" of momentum and energy and the effect of the shielding of one molecule by others, but does not consider the effect of multiple collisions. It is assumed as an approximation that, in a collision between nonrigid molecules, the collisional transfer takes place instantaneously at the time of closed approach. Expressions for the coefficients of shear and bulk viscosity and thermal conductivity are obtained in terms of a set of integrals which may be interpreted as generalizations of the "van der Waals' b" or covolume of the molecules. These integrals depend on the nature of the intermolecular potential.

From author's summary

3205. **Dutta, M., On a treatment of imperfect gases after Fermi's model (IV),** *Proc. nat. Inst. Sci. India* **17,** 6, 445–466, 1951.

Equations of state for gas mixtures have been derived by applying statistical methods to simplified models. Configuration space is divided into cells into which no more than one particle can enter. In this way the finite volume of the molecule is taken into account. Molecular interaction is taken into account by dividing configuration space into layers of different potential energy. For van der Waals' equation this amounts to the division of configuration space into an inner volume and a boundary layer. In this way the results of van der Waals, Lorentz, and Dieterici are duplicated. Reviewer doubts if the method can be adapted to more refined models. H. C. Brinkman, Indonesia

3206. **Nichols, J. B., An energy basis for comparison of performance of combustion chambers,** *Trans. ASME* **75,** 1, 29–33, Jan. 1953.

Author defines a single parameter "combustion effectiveness" to compare different combustion chambers. This definition includes only two variables, temperature rise and pressure drop, and is based upon maximum thrust produced for these given conditions. From this definition simple algebra produces an equation in terms of measurable quantities. Author prefers measuring thrust rather than exhaust temperature. The effect of high temperature dissociation is neglected.

One cannot argue about definitions but must accept them in the light of why such definitions are created. This simple definition is useful for combustion chambers whose purpose is to produce thrust. The algebra is accurate with but one minor misprint in the appendix. R. A. Gross, USA

3207. **Oppenheim, A. K., Water-channel analog to high-velocity combustion,** *J. appl. Mech.* **20,** 1, 115–121, Mar. 1953.

The hydraulic analogy to a unidimensional combustion system is derived. By a proper selection of state parameters, analogous relationships are derived to those between pressure and specific volume in a gaseous combustion system. The model for a stationary detonation could be achieved in a water channel. The combustion front discontinuity is simulated by a unidimensional source formed by admitting water from the bottom.

F. Schultz-Grunow, Germany

3208. **van Steenis, J., Determination of the velocity of burning,** *Nature* **170,** 4338, 1068–1069, Dec. 1952.

The author describes an optical method for measuring the velocity of burning of a gas flame. In order to measure deflection of the stream tubes through the flame cone, it is suggested that phase-contrast photographs rather than schlieren photographs are more suitable. References show that phase-contrast images, shifted 90 deg with respect to schlieren images, give gradients identical to those in the object. Measurements made inside and outside the cone are in better agreement from phase-contrast than from schlieren photographs.

H. M. Spivack, USA

3209. **Klein, G., and Prigogine, I., Statistical mechanics of irreversible phenomena. I** (in French), *Physica* **19,** 1–2, 74–88, Jan.-Feb. 1953.

Authors discuss critically the problem of calculating the distribution functions for nonequilibrium states, in particular the possibility of basing a statistical mechanics of irreversible processes on the recurrence relations between the distribution functions of various orders (Yvon, Born and Green) and the superposition principle of Kirkwood.

To this end, a particularly simple example of a linear assembly with interaction between nearest neighbors only is analyzed. This system has the advantage that its equilibrium properties are well known and the superposition principle is valid for a wide class of nonequilibrium states.

The equilibrium properties of this system are investigated in

detail, and, by virtue of the superposition principle, the recurrence relations are reduced to a single equation for the distribution function of couples of neighboring particles. The study of this equation for systems out of statistical equilibrium shows that other stationary solutions exist besides the equilibrium solution. L. M. Grossman, Holland

3210. Klein, G., and Prigogine, I., Statistical mechanics of irreversible phenomena. II (in French), *Physica* **19**, 102, 89–100, Jan.-Feb. 1953.

A continuation of preceding review, in which are studied several solutions of the equation for the distribution functions derived in part I for perturbations of the velocity distribution and energy partition.

The conclusion reached is that a method such as that of Born and Green, based solely on the recurrence relations and the superposition principle, is not suitable to describe dissipative processes; e.g., thermal conduction. It is shown, e.g., in the latter case that the inhomogeneous equation for the double distribution function of nearest neighbors, when a temperature gradient is present, has no stationary solutions nor solutions which admit a mean value in time. L. M. Grossman, Holland

3211. Cox, R. T., Brownian motion in the theory of irreversible processes, *Rev. mod. Phys.* **24**, 312–320, 1952.

The author discusses Brownian motion problems (with no effort to attain mathematical rigor), including the motion of a particle in a uniform force field, and of a harmonic oscillator. The treatment is based in part on an earlier paper [AMR **4**, Rev. 3698].

Courtesy of Mathematical Reviews J. L. Doob, USA

3212. Caskey, F. E., and Drickamer, H. G., Thermal diffusion in isotopic mixtures in the critical region, *J. chem. Phys.* **21**, 1, 153–156, Jan. 1953.

Thermal diffusion ratios have been measured for the system $C^{13}O_2$–CO_2 in the temperature range -15 C to $+50$ C and to 160-atm pressure. One isotherm at -15 C has also been measured for the system CH_4–CH_3H^3. The results deviate completely from the predictions of the kinetic theory of Enskog and Chapman and correspond closely to the previous results for the xenon-ethane system. The thermodynamics of irreversible processes qualitatively predicts the results obtained.

From authors' summary

3213. Pomp, A., Basic facts regarding the glowing of cold-rolled steel (in German), *Stahl u. Eisen* **73**, 3, 133–138, Jan. 1953.

This paper, which deals with radiations from cold-rolled steel and the corresponding metallographic and physical changes that take place, is very interesting and very timely. The change in properties with radiation time at various temperatures reveals very striking effects and indicates that, at temperatures of 800° and over, the effects are practically all dissipated in one or two minutes, whereas at lower temperatures the effects exist for several minutes and some effects exist for hours. The paper is very elaborately documented and refers to a total of 64 published papers, practically all of which are in German.

R. G. Sturm, USA

3214. Otte, H. M., and Ko, T., Thermal stabilization of austenite in carburizing steels, *J. Iron Steel Inst.* **173**, part 1, 31–35, Jan. 1953.

The quantity of retained austenite was determined in two carburized steels as dependent upon carbon content, austenitizing temperature, quench-bath temperature, and rate of cooling. The effect of holding at room temperature on stabilizing retained austenite was also studied. G. V. Smith, USA

3215. Rutter, J. W., and Chalmers, B., A prismatic substructure formed during solidification of metals, *Canad. J. Phys.* **31**, 1, 15–39, Jan. 1953.

Single crystals of tin grown from the melt under a wide range of conditions are shown to exhibit a fibrous structure which manifests itself as parallel ridges ("corrugations") on the free surface of the specimen and as a hexagonal network ("hexagonal cells") on the growing solid-liquid interface. The center of each cell projects into the liquid. Segregation of impurities is shown to occur during solidification in a manner intimately related to the structure. The structure is suppressed by growth at low speeds or under a steep temperature gradient. The size and regularity of the elements of the structure depend upon speed of growth, temperature gradient, and impurity content. A theory is advanced which accounts for the origin and observed properties of the structure. This theory shows that the structure consists essentially of a particular distribution of impurities resulting from nonequilibrium solidification.

From authors' summary by J. C. Fisher, USA

3216. Kelly, F. M., and MacDonald, D. K. C., On θ values in metals, *Canad. J. Phys.* **31**, 1, 147–164, Jan. 1953.

Following a discussion of specific heat theories, it is suggested that, despite its limitations, the Debye theory is the most useful representation for general analysis. Four methods of calculating θ_D, the Debye characteristic temperature, are presented; specific heat data of lithium, sodium, and potassium are then analyzed and also compared with similar calculations of θ_R from resistance measurements made over a wide temperature range. It is shown that more realistic values of θ are obtained when θ is recognized as a temperature-dependent parameter in the mathematical derivation of the relevant equations. It appears that a considerable amount of useful information may be obtained from data analysis in this way. From authors' summary

Heat and Mass Transfer

(See also Rev. 3122)

3217. Jakob, M., Relationship of heat transfer to mechanics and other branches of science, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 687–692, 1952.

Interesting relationships are indicated between heat transfer and thermodynamics, acoustics, electrophysics, light radiation, geometrical optics, and fluid mechanics. Only a few basic equations are utilized. Also included are brief remarks on the early developments in the field of heat transfer. L. Goland, USA

3218. Magenes, E., On the heat equation. Uniqueness and completeness theorems connected with Picone's integration method. Part I (in Italian), *R. C. Sem. Mat. Univ. Padova* 21, part I, 99–123, 1952.

The usual boundary-value problems for the heat equation assign (1) for almost all points of a cylinder with axis parallel to the time axis, the limiting values of the temperature for normal approach from the interior, or (2) for points on the curved surface of the cylinder, a linear radiation condition replaces the temperature in (1). Under rather mild analytical restrictions the author proves uniqueness of solution and completeness of certain sets of functions in terms of which a solution for arbitrarily shaped

boundaries may be obtained in principle. The method is adapted from that of Picone, to which a large literature is devoted. [A review is given by G. Fichera, *Mem. Accad. Lincei* (8) **3**, 1950.]

C. Truesdell, USA

3219. **Magenes, E., On the heat equation. Uniqueness and completeness theorems connected with Picone's integration method. Part II** (in Italian), *R. C. Sem. Mat. Univ. Padova* **21**, part I, 136–170, 1952.

The results of the foregoing paper (see preceding review) are extended to the case of mixed boundary-value problems.

C. Truesdell, USA

3220. **Sneddon, I. N., Solutions of the diffusion equation for a medium generating heat,** *Proc. Glasgow math. Assoc.* **1**, 21–27, 1952.

The heat equation in the form $\theta_t = k\nabla^2\theta + \phi(x, y, z, t) + \psi(t)\theta$, corresponding to the generation of heat within the medium at a rate that varies linearly with the temperature $\theta(x, y, z, t)$, is reduced to the form $u_t = k\nabla^2 u + \chi(x, y, z, t)$ by a change of variables. When the functions ϕ and ψ are prescribed, the boundary-value problem for the temperatures θ in an infinite solid whose initial temperature is a given function $\theta_0(x, y, z)$ is solved with the aid of three-dimensional Fourier transforms. The formula for θ is simplified by using the resultant or convolution property of that transform. Special cases are noted, including the case of axially symmetric flow of heat which is also treated by means of Hankel transforms. Other Fourier transforms are used to derive the formula for the temperatures $\theta(x, y, z, t)$ in a semi-infinite solid $x \geq 0$ when the face $x = 0$ is insulated and when the temperature of that face is a prescribed function of y, z, and t. In these cases, again the initial temperature is arbitrary and the rate of generation of heat within the body is the above linear function of the temperature.

Courtesy of Mathematical Reviews R. V. Churchill, USA

3221. **Blackman, J., The inversion of solutions of the heat equation for the infinite rod,** *Duke math. J.* **19**, 671–682, 1952.

Boundary-value problems for the heat equation [1] $u_{xx} = u_t$ and for the behavior of its classical solution

$$u(x, t) = (4\pi t)^{-1/2} \int_{-\infty}^{\infty} \varphi(\tau) \exp[-(x - \tau)^2/4t]\varphi(\tau)d\tau \quad [2]$$

are discussed, involving, in particular, extensions to complex values $z = x + iy$ of the first variable.

If A is a closed region with the square $0 \leq x \leq 1$, $0 \leq t < 1$ in its interior, and $u(x, t)$ is a solution of [1] continuous in the interior of A, then, under certain conditions on $u(0, t)$ and $u(1, t)$, the solution [2] may be extended to all z inside the square in the complex z-plane which has as diagonal the segment $0 \leq x \leq 1$, and $\lim_{t\to 1} u(z, t)$ exists and is analytic in the square.

If $F(\xi)(\xi = \sigma + i\rho)$ is analytic for $a \leq \sigma \leq b$, and

$$F(\xi) \leq g_1(\sigma) \exp[\rho^2 g_2(\sigma)]$$

where g_1, g_2 are bounded in every finite subinterval of (a, b), and T positive and sufficiently small, then there is a $u(x, t)$ of form [2] such that $F(\xi) = \lim_{t\to T} u(\xi, t)$, $a \leq \sigma \leq b$. Tychonoff [*Mat. Sbornik* **42**, 199–216, 1935] showed that if the integral [2] converges for x_0, t_0, it converges for all x and t with $0 \leq t < t_0$. Examples are constructed in which [2] converges for $t < t_0$ but not for $t = t_0$, while $u(x, t)$ can be extended beyond $t = t_0$ to give a solution of [1] in the region of extension. An application is given to inversion of a Weierstrass summation of a Fourier integral.

Courtesy of Mathematical Reviews J. L. B. Cooper, Wales

3222. **Krueger, M., On the combination of thermic and electromagnetic fields of two-dimensional plates** (in German), *Ing.-Arch.* **20**, 234–246, 1952.

The control of temperatures in a plate in which heat is generated by a high-frequency alternating current is examined theoretically. Let $\theta(z, t)$ denote the temperatures within a plate of infinite extent whose faces are the planes $z = 0$ and $z = d$. Heat transfer, according to Newton's law of cooling, takes place at those faces into media whose temperatures are prescribed functions of the time t. The initial temperature of the plate is a prescribed function of z. Let another function of z represent the rate of generation of heat within the plate; thus $\theta_t = a^2\theta_{zz} + f(z)$ when $0 < z < d$ and $t > 0$. This boundary-value problem is solved for $\theta(z, t)$ with aid of the Laplace transformation. The author then shows how the function $f(z)$ or some of the prescribed functions, the initial temperature or the temperatures of the adjacent media, can be determined so that at a given instant $t = t_0$ the temperatures in the plate will be a prescribed function $g(z)$. In particular, the source function $f(z)$ is determined so that $g(z) = \theta_0$, a constant, when the initial temperature is zero and the faces are kept at the temperature θ_0. When the plate is a dielectric and the source function $f(z)$ arises from a quasi-stationary electromagnetic field, it is shown that, under certain restrictions, $f(z) = k_1 e^{kz}$, where k_1 and k denote constants. In this case, a uniform plate temperature θ_0 at $t = t_0$ can be obtained by adjusting initial and boundary conditions only when $k = 0$.

Courtesy of Mathematical Reviews R. V. Churchill, USA

3223. **Roche, E., Application of the motion of relative heating to the study of forced convection** (in French), *Publ. sci. tech. Min. Air, Paris* no. 267, 77 pp., 1952.

Transfer of heat during flow of fluids through tubes, past flat surfaces and across tube banks is discussed thoroughly from the point of view of relative heating, i.e., temperature rise of fluid with respect to temperature driving force, and the advantages of this viewpoint are presented. Simple generalized equations for heat transfer and pressure drop, including entrance effects, are developed for viscous and turbulent flow. A limited amount of experimental data is reported for viscous flow through tubes of an oil cooler, and for flow of air past aircraft engine cylinders and three types of extended surface radiators. The data are compared with values calculated by recommended equations.

O. P. Bergelin, USA

3224. **Hamilton, D. C., and Morgan, W. R., Radiant-interchange configuration factors,** *NACA TN* 2836, 110 pp., Dec. 1952.

This report is concerned with the geometric configuration factor for computing radiant interchange between opaque surfaces separated by a nonabsorbing medium. The configuration-factor solutions available in the literature have been checked and the more complicated equations are presented as families of curves. Several new configurations involving rectangles, triangles, and cylinders of finite length have been integrated and tabulated. The various methods of determining configuration factors are discussed, and a mechanical integrator is described. An analysis is presented, in which configuration factors are employed, of the radiant heat transfer to the rotor blades of a typical gas turbine under different conditions of temperature and pressure.

This work represents a welcomed extension of currently existing configuration factor charts. W. M. Rohsenow, USA

3225. Sauer, F. M., and Drake, R. M., Jr., Forced convection heat transfer from horizontal cylinders in a rarefied gas, *J. aero. Sci.* **20,** 3, 175–180, 209, Mar. 1953.

The average heat-transfer coefficient from a horizontal cylinder to a flowing continuous fluid was calculated theoretically, agreeing with accepted experimental values. In a gas sufficiently rarefied that the mean free path is comparable to the diameter of the cylinder, temperature jump occurs at the surface. With a simplifying assumption for free-stream velocity distribution around the cylinder, local and upstream average values of Nusselt number are computed for different positions on the surface. Good agreement is obtained for Mach numbers near 2 with the trend of experimental data, and moderate agreement on the absolute magnitudes. C. F. Bonilla, USA

3226. Mattarolo, L., Experiments on the heat transfer in flowing air at low temperatures (in Italian), *Ric. sci.* **23,** 1, 81–97, Jan. 1953.

Heat-transfer coefficients are determined for the pipe flow of air at temperatures between 0 and −70 C under normal and high pressures up to 200 atm. Reynolds number Re is based on tube diameter ranges from 4×10^3 to 5×10^4. Experiments at normal pressure agree with the Nusselt-type formula $Nu/RePr = 0.023\text{-}Re^{-0.2}Pr^{-0.7}$, but heat transfer at high pressure is 10 to 20% higher than the value given by this formula.

H. Tamaki, Japan

3227. Wilson, W. E., and Mitchell, W. I., Self-induced temperature effects on laminar flow of liquids, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 789–795, 1952.

Author considers laminar rectilinear flow of a liquid between two parallel plates, one of which may have a constant translational velocity with respect to the other (or may be at rest), or through a cylindrical pipe. The work done by the tangential force on the moving wall plus the work done by the pressure drop is converted into heat and produces a rise of temperature. There is no loss of heat through the walls. The change of viscosity with temperature is calculated from $\mu = \mu_0 e^{-\beta T}$ instead of $\mu = \mu_0 e^{\beta/T}$ (which would be preferable from a physical point of view). The time-independent problem is considered (leading to a singularity when the total flow is zero with pressure gradient acting opposite to moving wall). The results, given in diagrams for certain nondimensional variables, are discussed.

J. M. Burgers, Holland

3228. Ingebo, R. D., Study of pressure effects on vaporization rate of drops in gas streams, *NACA TN* 2850, 36 pp., Jan. 1953.

The rate of evaporation of four pure liquids (n-octane, benzene, acetone, and methanol) was studied experimentally by exposing a porous sphere (between 0.7 and 0.8 cm in diam), which was kept saturated with the liquid under study, to a carefully controlled gas stream. The sphere and the vessel supplying it with liquid were weighed before and after each exposure. The gas in most cases was air, with temperatures ranging from 22 C to 93 C, and various wind speeds. The gas pressure was varied between 450 and 1500 mm of Hg.

The temperature of the gas stream and of the surface and the interior of the porous sphere was measured by thermocouples. By maintaining the temperature of the liquid supplied to the "drop" close to that of the surface, heat transfer through the liquid was kept negligible in comparison to the latent heat of vaporization.

Kinetic theory considerations lead to the formulation of an equation suitable for the description of the process, notably the effect of pressure variation. The observations can be expressed with reasonable accuracy by an equation for h, the heat-transfer coefficient [cal cm^{-2} sec^{-1} (deg temp. difference)$^{-1}$] for a spherical drop

$$h = \frac{2k_g}{d}\left[1 + 1.29 \times 10^6 \left(Re\, Sc\, \frac{gl}{\bar{c}^2}\right)^{0.6} \left(\frac{k_g}{k_v}\right)^{0.5}\right] \quad [1]$$

where k_g and k_v are the thermal conductivities of the gas and the vapor, d is the drop diameter, Re and Sc are the Reynolds and Schmidt numbers, g is the acceleration due to gravity, l is the mean free path of the gas molecule, and $\bar{c}$ is their root-mean-square velocity. The product $Re\ Sc = du\rho/b_{g,v}$ with u the gas velocity, ρ the gas density, and $b_{g,v}$ the molecular mass diffusivity of the vapor ($= PM_vD_v/RT$, where P is the pressure, M_v the molecular weight of the vapor, and D_v its diffusivity, R the universal gas constant, and T the absolute temperature; units are g cm^{-1} sec^{-1}).

The heat-transfer coefficient given by Eq. [1] determines the rate of evaporation according to the heat-balance equation

$$dm/d\theta = (hA/H_v)\Delta t \quad [2]$$

where m is the mass of liquid, θ the time, H_v the latent heat of vaporization, A the surface area of the drop, and Δt is the difference between the temperature of the gas and that of the drop surface.

Eq. [2] does not permit the calculation of the rate of evaporation unless Δt is also known. This quantity must be measured, or else may be obtained (at least for some liquids) from psychrometric information. Indirect calculation of Δt (by combining Eqs. [1] and [2] and the diffusion equation) was not attempted in this paper, and is, in any case, not a really satisfactory procedure.

One important property of h which may be deduced from Eq. [1] is its independence of the pressure. The rate of evaporation $dm/d\theta$ thus depends on the pressure only through Δt. A special examination of this effect was made for water for which psychrometric data are quoted (though their source is not given; rather a serious omission, considering the difficulties of making measurements at temperatures and pressures way above the boiling point). From these, author deduces that for temperatures of 100 C or higher

$$\Delta t = \Delta t^* - 33 \log P \quad [3]$$

where Δt^* is the value of Δt at a pressure of 1 atm. Here P may range from 0.2 to 2 atm.

Reviewer considers article an interesting contribution to literature on heat and mass transfer from liquid drops. He regrets, however, the involved and, in part, far-from-standard notation, notably the excessive use in tables and graphs of dimensionless groupings. Such groups of parameters are valuable in the final presentation and for generalizations, but they obscure the essential physics of the situation. W. Hitschfeld, Canada

3229. Bromley, L. A., Brodkey, R. S., and Fishman, N., Heat transfer in condensation. Effect of temperature variation around a horizontal condensate tube, *Indust. Engng. Chem.* **44,** 12, 2962–2966, Dec. 1952.

Temperature variation of wall of condenser tube is calculated for two cases, one with infinite tube-wall thermal conductivity, and one with zero conductivity. The one with infinite conductivity fulfills Nusselt's assumption of constant tube-wall temperature in calculation of heat-transfer rate. Maximum deviation of ratio of temperature drop through wall, scale, and cooling film to total drop is 0.8%, but a possible error of 6.5% (Nusselt value high) in individual coefficient is indicated.

Experimental data on stainless-steel tube and on Monel-metal tube agree with calculated temperature variation for zero wall conductivity except near bottom of tube, where large departures from calculated values are attributed to formation of liquid into drops rather than in thin sheet of infinite length.

Conclusion is that Nusselt equation should be used for design, since calculated and experimentally determined deviations are negligible. B. E. Short, USA

3230. **Bromley, L. A., Effect of heat capacity of condensate,** *Indust. Engng. Chem.* **44,** 12, 2966–2969, Dec. 1952.

"For organic vapors at high pressures and large temperature differences between saturated vapor and tube temperature, contribution of sensible heat term may significantly affect heat transfer."

A modification of the Nusselt equation permits calculation of the effect of heat capacity of condensate on heat-transfer coefficient. The modified equation is satisfactory for values of $\Delta t C_p/\lambda$ up to about 3.0. The same type of equation applies to film boiling. At this value of $\Delta t C_p/\lambda$, the ratio of heat-transfer coefficient calculated with modified equation to that from the Nusselt equation is about 1.5. For usual designs for steam and refrigerant (condensers) vapors ($\Delta t C_p/\lambda < 0.5$), the ratio would be 1.1 or less. B. E. Short, USA

3231. **Frenkel, M. S., The "counter-flow" evaporator element; its operation and applications,** *Engng. and Boiler House Rev.* **66,** 7, 198–205, July 1951.

This paper is an abridgement of an original article by Dr. Frenkel; the original was abridged by P. Meyer.

The paper describes a new type of evaporator element which has application in any evaporation process which can be handled inside vertical tubes. The advantages claimed for this evaporator element over conventional designs are: (1) Strict separation of vapor and liquid without the use of external separators, and, at the same time, at least as great a vapor production as with conventional evaporators of the same tube size; (2) a positive driving force (greater than gravity head) causing liquid circulation; (3) possible utilization of the kinetic energy of the exit vapor in contrast to the loss of kinetic energy in conventional liquid-vapor separators; (4) flexibility of operation.

The term "counter-flow" is used, not in its usual sense (heating medium flowing opposite to cooling medium) but rather to indicate that the substance being evaporated reverses its direction of flow after being vaporized. Hence, the vapor flows counter to the unevaporated liquid, both within the evaporator tube.

The evaporator consists of an externally heated vertical tube. A smaller tube (or nozzle) intrudes concentrically at the top of the evaporator tube and withdraws vapor from the core of the evaporator tube, carrying that vapor to the vapor receiver. Liquid flows into the top of the evaporator around the intruding vapor tube, thus setting up annular flow of the liquid down the evaporator tube wall. The paper shows theoretically that the flow pattern maintains itself; that is, downward annular flow of liquid and upward central flow of vapor will persist. Furthermore, the paper shows that, in theory, this type of flow results in the advantages claimed.

An interesting application discussed is the use of this evaporator element in place of the normally nonevaporating liquid downcomers of steam boilers. In this case, the element provides additional evaporating surface, as well as an increase in the force circulating the liquid, and a quantity of dry vapor.

No experimental or operating data are included.

S. Zivi, USA

3232. **Verö, J., A precise method of measuring temperatures for thermal analysis** (in Hungarian), *Öntöde* **2,** 4, 80–84, 6 figs., 1 tab., Apr. 1951.

Article deals with thermocouple measuring and points to the shortcomings of the connection wiring and of the galvanometer. The resistance of the latter two is subject to changes, and, since even the zero point of the galvanometer shifts, the read values do not correspond with the actual temperature. To overcome this drawback, galvanometers have been designed for greater resistances. However, there is no remedy against the changed resistance of the thermocouples due to annealing. Temperature measuring by potentiometer can be considered more accurate. Special reference is made to Hungarian literature in this connection. Wiring diagrams are furnished and the operating principle of the potentiometer is described. Several calibration tests were made by the author for measuring the melting point of pure metals. The description of the measuring method extends to error sources as well. In conclusion, measuring results and formula for their evaluation are presented.

Courtesy of Hungarian Technical Abstracts

Acoustics

(See also Rev. 3002)

3233. **Keller, J. B., Finite amplitude sound waves,** *J. acoust. Soc. Amer.* **25,** 2, 212–216, Mar. 1953.

Exact periodic solutions of the nonlinear one-dimensional gas dynamical equations are obtained for an inviscid polytropic fluid with adiabatic exponent $\gamma = -1$ (Chaplygin gas). For this medium (liquid or gas), the exact equations reduce to the ordinary linear wave equation in appropriate Lagrangian coordinates. Two particular problems are solved in detail: (1) The progressive sound wave radiated by a sinusoidally oscillating piston; (2) the free stationary wave in a closed tube. Graphs of particle velocity and specific volume as functions of both position and time are plotted for various amplitudes. No shocks will occur in this medium if the amplitude of the sound wave is less than the wave length divided by 2π. R. Heller, USA

3234. **Torikai, Y., and Negishi, K., The application of the phase method in visualizing ultrasonic waves,** *J. phys. Soc. Japan* **8,** 1, 119–124, Jan.-Feb. 1953.

Ultrasonic waves propagating in a transparent medium can be considered optically as a sort of phase grating. Therefore, the method of phase microscopy may be applied to the image formation of ultrasonic waves. The application of the phase method is studied theoretically and experimentally in visualizing ultrasonic waves, and authors confirm the effect of the method for the improvement of the visibility of the image. In the usual schlieren method for visualizing ultrasonic waves, the zero-order diffraction image is perfectly screened, while in authors' method the image receives a proper absorption and phase retardation by the phase plate placed in the diffraction plane. According to the theoretical calculations, the phase retardation for the maximum visibility of the image of progressive waves with the stroboscopic illumination is $\pi/2$, and the optimum absorption varies with the intensity of the waves. In both cases of the usual method and the present phase method, the theoretical curves are compared with the experimental results, showing a considerable agreement between them. Further, the fine structures in the images are seen with the increase of sound intensity. From authors' summary

3235. Petralia, S., Velocity and absorption of ultrasonic waves in gases (in Italian), *Nuovo Cim.* (9) **9,** Suppl. 1, 1–58, 1952.

Paper reviews knowledge about propagation speed of ultrasonic waves and their absorption by gases. As the classical wave theory does not furnish results for ultrasonic intervals in accordance with experience, author studies various influences (specific heat of gases, oscillation molecular heat, etc.) and also their consequences on dispersion bands. He deals also with measuring of absorption speed in gases, based on interferometric measurements, relates the experimental result on biatomic, triatomic, and polyatomic gases, and gives two methods, dynamical and optical, for measuring relaxation time. Bibiography is very complete, numbering 178 references. L. Villena, Spain

3236. Storey, L. R. O., Ultrasonic absorption in mixtures of ethyl alcohol and water, *Proc. phys. Soc. Lond.* (*B*) **65,** part 12, 396B, 943–950, Dec. 1952.

The pulse method has been used to measure the absorption of ultrasonic waves in mixtures of ethyl alcohol and water at the three frequencies, 22.5, 37.5, and 52.5 mcps and at the temperatures 0, 10, and 25 C. The results confirm the findings of earlier workers that in both the pure liquid and the mixture there is more absorption than can be explained by the classical mechanisms of viscosity and heat conduction. When the composition of the mixture is varied, a maximum of absorption is found for a mixture containing 20 to 30 molar per cent alcohol. The magnitude of the absorption at the maximum increases rapidly as the temperature is lowered. Over a range of compositions in the neighborhood of the maximum, the value of α/f^2 (α is the absorption coefficient and f the frequency) is found to decrease as f increases.

It is suggested that the additional absorption is due to interaction between the molecules of the two components, leading to the partial development of a structure or complex in the liquid, with a composition of four molecules of water to one of alcohol. The thermal properties of the mixture indicate that such interaction occurs, and the suggestion for the composition of the complex is supported by the fact that the freezing-point curve for the mixture shows that a structure with this composition is formed in the solid phase. The extra absorption found by other workers in several similar mixtures can be explained, at least qualitatively, in the same way. From author's summary

3237. Parker, J. G., Adams, C. E., and Stavseth, R. M., Absorption of sound in argon, nitrogen, and oxygen at low pressures, *J. acoust. Soc. Amer.* **25,** 2, 263–269, Mar. 1953.

The amplitude absorption coefficient for pulsed sound waves in argon, nitrogen, and oxygen has been measured over a range of pressure from 1 to 10 mm Hg for frequencies between 60 and 70 kc, the temperature in all cases being held nearly constant at 20 C. For the three gases used, the following experimental values of $(p\alpha/f^2)\ 10^7$ (cgs units) were obtained: (1) for argon, 1.86 with an rms deviation of 0.03; (2) for nitrogen, 1.64 with an rms deviation of 0.04; and (3) for oxygen, 1.92 with an rms deviation of 0.03. The corresponding values computed from the classical absorption equation are (1) for argon, 1.87; (2) for nitrogen, 1.31; and (3) for oxygen, 1.61. Thus the absorption in argon is classical, while for nitrogen and oxygen the absorption exceeds the classical value. These excesses are attributed to rotational relaxation, and the associated relaxation times are calculated in accordance with the thermal relaxation theory of Herzfeld and Rice, and Kneser. For nitrogen, the relaxation time (reduced to STP conditions) is 4.85×10^{-10} second, and for oxygen is 4.95×10^{-10} second. Both these values are significantly smaller than the values obtained by other workers at higher frequencies.

From authors' summary

3238. Kurtze, G., and Tamm, K., Measurements of sound absorption in water and in aqueous solutions of electrolytes, *Acustica* **3,** 1, 33–48, 1953.

Absorption over large frequency range, 4 kc/s to 100 mc/s, was measured using four methods. With water, the characteristic constant α/ν^2 was $25 \times 10^{-15}\, s^2/M$ at 20 C, independent of frequency down to 100 kc/s. With the electrolytes, the results indicated relaxation processes; four models are discussed: (1) Excitation of molecular degrees of freedom; (2) hydration of the ions; (3) formation of ion associates; and (4) hydrolysis (the most probable).

From authors' summary by P. H. Parkin, England

3239. Jones, D. S., A simplifying technique in the solution of a class of diffraction problems, *Quart. J. Math., Oxford Ser.* (2) **3,** 189–196, 1952.

It is well known that a class of diffraction problems gives rise to integral equations of the Wiener-Hopf type. The author shows that one may by-pass the integral equation by applying the transform in question directly to the partial-differential equation. After a certain amount of manipulation, one is brought back to the same function-theoretic problem that one encounters in the original Wiener-Hopf work. Such views were expressed in part by M. Kantorovich and N. N. Lebedev [*Akad. Sci. USSR J. Phys.* **1,** 229–241, 1939]. A. E. Heins, USA

3240. Kawai, N., On the theory of Rayleigh disc, *Sci. Rep. Tôhoku Univ.* (I) **35,** 210–231, 1952.

In solving the problem of the diffraction of a plane acoustic wave by a circular disk, the author uses an expression for the velocity potential consisting of a double Fourier series, the terms of which contain infinite integrals involving the products of two Bessel functions of the first kind in their integrands. The circular disk is assumed to vibrate at the same frequency as the incident wave, the vibration being translational and rotational. From the boundary conditions on the disk's surface the coefficients of the above series are obtained. From this the total pressure on the disk is calculated. The resulting torque on the disk is then evaluated. Numerical values for these two expressions are then given up to $k = 2\pi a/\lambda = 4$, where a is the disk's radius and λ the wave length in the surrounding medium at different angles of incidence. Two appendixes and numerous tables and sets of curves complete the paper. Viscosity and turbulence are not accounted for.

Courtesy of Mathematical Reviews M. J. O. Strutt, Switzerland

3241. Miles, J. W., On acoustic diffraction through an aperture in a plane screen, *Acustica* **2,** 6, 287–291, 1952.

The scattered wave consequent on the incidence of a monochromatic disturbance on a plane screen perforated by an aperture is constructed with the aid of generalized Fourier transforms. After formulating the integral equation defining this scattered wave, the total transmission cross section is cast in variational form (of the Schwinger type) and the result compared with the Kirchhoff approximation. A similar result is given for scattering by a plane disk. Application is made to a circular aperture.

From author's summary

The complementary optical problem is treated in a more extensive form by Levine and Schwinger [*Phys. Rev.* **74, 75,** p. 958, p. 1423; 1948, 1949]. G. Moretti, Argentina

3242. Gerjuoy, E., Total reflection of waves from a point source, *Comm. pure appl. Math.* **6,** 1, 73–91, Feb. 1953.

A mathematical analysis of the reflection of a pulse of sound from a harmonic point source in a nonabsorbing liquid at a plane

boundary of indefinite extent, the nonabsorbing medium 2 beyond the interface having a smaller index than the source medium 1. Sound field in medium 2 has been discussed previously by author [AMR 2, Rev. 8]. To obtain sound field in medium 1, the exact solution of the wave equation is evaluated in the complex plane by means of the method of steepest descents, extending the work of H. Ott [*Ann. Physik, Leipzig* **41,** 6, 443–466, July 1942] and L. Brekhovskikh [*Zh. tekh. Fiz. USSR,* **18,** 455–472, Apr. 1948 (in Russian)]. A simple geometrical interpretation is given of the amplitude dependence of the head wave, sometimes observed in geophysical prospecting and in underwater propagation. Also studied is the contribution which a pole lying on the imaginary axis in the complex plane of integration may make to the field in medium 1.

The analysis is equally applicable to the total reflection of electromagnetic radiation from a dipole perpendicular to the interface. R. Heller, USA

3243. Swartzel, K. D., A review of the problem of external aircraft noise, *Aero. Engng. Rev.* **12,** 3, 49–54, Mar. 1953.

Physical and psychological problems created by loud noises associated with modern aircraft are of major importance to engineers associated with aircraft industries and the armed services. Author gives a brief, extremely simplified, review of six phases of the problems; physics of sound, the ear as a detector of sound, psychological considerations, mass psychology, politics, and economics. Reviewer believes the article is an excellent introduction for those unfamiliar with these problems, but is of little value to anyone active in the field of aircraft noise.

E. Ackerman, USA

3244. Hayhurst, J. D., Acoustic screening by an experimental running-up pen, *J. roy. aero. Soc.* **57,** 505, 3–11, Jan. 1953.

There is much current interest in practical methods for the reduction of noise during ground testing of airplane engines. The present paper describes some studies made at London airport to evaluate the usefulness of an experimental wall in shielding adjacent areas from jet and propeller noise. A 40-ft high wall of corrugated cement-asbestos sheeting, fixed to a scaffold of steel tubing, gave a reduction of approximately 25 phons throughout most of its geometrical shadow and for a range of distance exceeding one mile. The noise reductions thus observed were many times greater than those caused by a 100-ft wide belt of trees.

H. H. Hubbard, USA

Ballistics, Detonics (Explosions)

3245. Blanc, P., Mathematical analysis of multistage rockets (in French), *Mém. Artill. fr.* **26,** 2, 706–734, 1952.

Paper considers some of the theoretical and practical benefits to be derived from a rocket construction consisting of several motors acting successively, which are gradually released one by one—a principle known as "multistage" disposition.

With one-stage rockets, performance is limited according to the possibilities of obtaining the ejection velocity and the proper mass-ratio. Application of the multistage principle permits obtaining higher ratio of masses and, consequently, better performance than would ever be possible by using a single-stage rocket. For a multistage rocket construction, a mathematical solution of the problem is here presented. After a discussion of the simple rocket, author includes analysis of the two-stage rocket and then comes to the general case of multistage rockets.

The optimal solution for construction of a multistage rocket can be found with such a weight of each motor that the mass ratio of each stage is proportional to the ratio of the mass proper of the corresponding motor.

Theoretically, the best performance can be obtained by using a great number of stages or propulsors, but there is a limit set by the allowable dimensions of the rocket as a whole. The best practical solutions could be obtained only by augmenting ejection velocities. J. Marinković, Yugoslavia

3246. Engdahl, G., Valuation of anti-aircraft guns (in Swedish), *Artill. Tidskr.* **81,** 6, 191–200, 1952.

As a contribution to the running discussion about anti-aircraft guns, author compares three guns: the new remote-controlled 40-mm Bofors gun, the new remote-controlled 57-mm Bofors gun, and a supposed gun, called X-gun, with subcaliber ammunition.

Starting from the effect of a single shot in targets of different weights and the probability of a hit in the target, author makes a relative comparison between the different guns from the economic viewpoint as well as in the form of effect per time unit. In brief, the conclusions are: (1) The most economical anti-aircraft weapon against targets with weights up to 10–12 ton within ranges of 4–5 km is the 40-mm gun. (2) Against heavier targets than 10–12 ton, the 57-mm gun is more economical than the 40-mm gun in ranges over 2–3 km. (3) The supposed X-gun is superior to the other guns against targets heavier than 10–15 ton within all ranges over 2–3 km.

Author calls attention to the uncertainty in the assumptions, but insists upon the right tendency of the conclusions.

R. Sjöberg, Sweden

3247. Triponez, A., Aiming problems in anti-aircraft artillery (in French), *Flugwehr und Technik* no. 12, 272–276, Dec. 1952.

3248. Schauer, H. M., The afterflow theory of the reloading of airbacked plates at underwater explosions, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 887–892, 1952.

Author investigates the flow through a circular orifice under the action of a source in an incompressible fluid as a model for the phase of underwater explosion near a plate which follows the initial shock loading and reloads the plate after cavitation has set in. The source strength history is adjusted to simulate the history of the expanding explosive globe, and the stress in the plate is estimated by equating the kinetic energy of water motion (approximately adjusted for edge effects) to the energy of plastic deformation of the plate. The agreement with tests on rectangular plates is better than qualitative. L. Trilling, USA

Soil Mechanics, Seepage

(See also Rev. 3113)

3249. Tiedemann, B., On investigations of soil properties in design and construction of engineering projects [Über Bodenuntersuchungen bei Entwurf und Ausführung von Ingenieurbauten], Berlin, Ernst & Sohn, 1952, 72 pp., 33 figs., 7 tables. DM 8.50.

This is the fourth edition, revised by K. v. Bülow after the death of the author in 1945. The first edition appeared in 1941 and the third in 1946.

Introductory material is followed by a short chapter on exploratory field investigations—pits, bore-holes, and water-level observations. The third chapter deals with the classification of soils and rocks on the basis of their mechanical and physical properties which are briefly outlined. The fourth chapter describes tests in-situ and in field laboratories, and their evaluation.

Instruction sheets and classification tables are joined to the text, summarizing present German practices.

G. P. Tschebotarioff, USA

3250. Bernhard, R. K., Static and dynamic soil compaction, Highway Res. Bd. Proc., 31st ann. Meet., 563–591, 1952. $7.50.

Paper describes results of extensive investigations to compare effects and to determine efficiencies of static and dynamic compaction methods. Static loads were produced by rollers, tractors, or dynamic compactors with dynamic unit not operating. Four types of dynamical compactors were used, covering a wide range in total weight, dynamical force and action, frequency, etc. Investigations were made using six types of soils in three confined (4-in. diam Proctor mold, 25.75-in. diam container, and 11 × 24 × 4 ft basin) and two unconfined (highway subbase with concrete pavement and without pavement) conditions. Compactive effects were determined by measuring soil density, surface settlement, displacement amplitude, and pressure transfer (using pressure cells).

Results show that combined static and dynamic loads compact deeper strata more effectively than static compaction or dynamic compaction alone. Indications are that height of layers to be compacted may be increased to 2 ft or more. Self-propelled dynamic compactors are recommended for compaction of confined areas and units not self-propelled for unconfined areas; in both cases, frequencies from 15 to 40 cps, adjustable to the critical frequency of the vibrating system (similar to resonant frequency for the compactor-soil combination) were found to be advantageous. While results are indicative, they should not be viewed as exhaustive but as preliminary results in a more comprehensive program.

G. J. Tauxe, USA

3251. Schultze, E., Resistance of soil foundations to oblique base pressure (in German), *Bautechnik* **29,** 12, 336–342, Dec. 1952.

Using the theory of Buisman-Caquot, the author calculates the bearing power of foundations for the case that the resultant of the foundation loads is not perpendicular to the loaded area. The same was done earlier by Etienne Robert [AMR **3,** Rev. 1181]; however, the writer has considered the influence of the weight of the soil under foundation level.

Equations are applied to centrically and eccentrically loaded foundations. An example is given. F. C. de Nie, Holland

3252. Childs, E. C., The measurement of the hydraulic permeability of saturated soil *in situ*. I Principles of a proposed method, *Proc. roy. Soc. Lond.* (*A*) **215,** 1123, 525–535, Dec. 1952.

Proposed method for in-situ determination of the permeability of water-bearing soil is described in detail. Technique involves the application of steady-state fluid-flow analysis to a two-well system. Water is pumped from one well to the other at a known constant rate until steady-state conditions obtain. The difference of water levels in the injection and producing wells is related to the rate of water flow and the horizontal permeability of the soil between the two wells. By proper orientation of two pairs of wells, the horizontal directional permeabilities of anisotropic media may be determined. Combined with the method of D. Kirkham [*Proc. Soil Sci. Soc. Amer.* **10,** 58, 1945], this method permits the calculation of the axial permeabilities in a triaxially anisotropic medium. Of interest is the utilization of two-dimensional electric analogs for the solution of the two-well system in anisotropic media. Reference should be made to the analysis of steady-state flow for a two-well system by M. Muskat ["The flow of homogeneous fluids through porous media," McGraw-Hill Book Co., Inc., New York, section 4.7, chapter IV, 1937]

S. R. Faris, USA

3253. Childs, E. C., Cole, A. H., and Edwards, D. H., The measurement of the hydraulic permeability of saturated soil *in situ*. II, *Proc. roy. Soc. Lond.* (*A*) **216,** 1124, 72–89, Jan. 1953.

Method proposed by Childs (see preceding review) for measurement of the hydraulic permeability has been checked experimentally by study of steady-state flow phenomena occurring between two wells situated in a tank of sand of substantially homogeneous character. Comparison of experimental results with theory permits the conclusion that errors introduced by perturbation of the water table level resulting from the difference between water levels in two wells are negligible, even when this difference may amount to as much as 30% of the undisturbed water depth. The magnitudes of errors arising from end effects produced by wells partially penetrating the sand bed, and from capillary phenomena occurring at the free water surface in the sand bed are evaluated. Method of correcting for these errors is presented for both isotropic and anisotropic beds. A preliminary field test of this method of permeability measurement is briefly described and results are presented. S. R. Faris, USA

3254. Lane, D. A., Torchinsky, B. B., and Spinks, J. W. T., Determining soil moisture and density by nuclear radiations, *Engng. J., Montreal* **36,** 1, 1–7, Jan. 1953.

The development of a meter which can be used to determine soil moisture and density is described. The moisture meter is based on fast neutron bombardment of the soil, causing reflection of slow neutrons which activate a detector foil. The activity of the detector foil after a given exposure time can be related to the water content of the soil. The density meter, using the same source and holder as the moisture meter, measures with a Geiger tube the gamma rays reflected by the mass of material through which they radiate. The zone of influence can be considered to be a sphere of about a 6-in. radius. Both the moisture meter and the density meter give a linear calibration.

Field testing of the moisture and density meters on sandy silt and medium plastic alluvial clays yield results which agree within 3% moisture, and most within 2% moisture, or less. Less accuracy was obtained when testing Bearpaw shale, the neutron meter indicating more moisture than is actually present. The presence of large amounts of organic material would also upset the calibration curve. Although considerable work is still necessary to determine the effect of these and other factors, results obtained in preliminary tests indicate that the neutron meter is of considerable practical value.

Reviewer believes that, since soil-moisture determination is a major bottleneck in many aspects of engineering investigation, the development of new tools such as that indicated herein is most encouraging. V. E. Hansen, USA

3255. Mallet, Ch., and Pacquant, J., Methods and apparatus for the measurement of deformation in earth dams and their foundations (in French), *Trans. Fourth Congr. Inter. Comm. Large Dams*, New Delhi, Jan. 1951, in 4 vols., vol. I, 303–327. $48 per set.

After a brief review of the reasons for measuring deformations in earth dams and their foundations, the authors discuss the methods used in investigating the behavior of the 27-meter high Sarno Dike.

The apparatus which they developed are believed to be strong and reliable enough to allow their use during placing operations. They were developed in order to follow the deformations during the uncovering of the foundation and during the placement of the

rolled-fill material. Two types of apparatus were employed, one for measurements in the foundation and the other for inaccessible points in the mass of the dike itself. The article describes both of these devices as well as giving schematic diagrams of them. Both of them are based on simple principles and the devices are not very complicated.

Attention is drawn by the authors to the fact that the use of the "tele-level," used for measuring within the mass itself, requires special precautions to avoid gross errors. Little attention is given to the interpretation of the results obtained, the article being more descriptive of the apparatus than analytical in nature.

J. R. Bruggeman, USA

3256. Grishin, M. M., Design and construction of earth and rock-fill dams in the U.S.S.R., *Trans. Fourth. Congr. Inter. Comm. Large Dams*, New Delhi, Jan. 1951, in 4 vols., vol. IV, 579–616. $48 per set.

"This paper deals with the peculiarities of designing and constructing earth and rock-fill dams in the U.S.S.R." Theoretical principles for seepage in dams and stability of dams are reviewed, with many Russian references included. Design formulas are developed for seepage, capillary water, infiltration of rain, and consolidation.

Construction procedures are described, some of which are unique. These include methods of building frozen earth dams, dry-placed fills without cofferdams or dewatering, and hydraulic dams of sand without cores. Practice has shown that hydraulic dams may be built during winter when temperature is below −15 C. Investigations of deformations in rock-filled dams are included.

Reviewer believes this paper gives the state of Russian knowledge and achievement in the field of dam construction.

J. A. Cheney, USA

3257. Winqvist, G., Ground water in Swedish eskers, *Trans. roy. Inst. Technol., Stockholm* no. 61, 91 pp., 1953.

Paper presents a discussion of three important eskers in Sweden which are sources of ground water for municipal water supply. The formation, extent, and hydraulic continuity of the eskers are discussed and estimates made of the available ground-water supply in each. Detailed calculations of water supply are presented in appendixes. Paper is of interest generally in showing methods of approach in estimating available water supplies from limited underground sources such as eskers.

W. G. Shockley, USA

3258. Dietz, D. N., A theoretical approach to the problem of encroaching and by-passing edge water, *Proc. k. Ned. Akad. Wet.* **(B) 56,** 1, 83–92, Jan./Feb. 1953.

Author gives a clear picture of various phenomena encountered in oil production from underground reservoirs, with special reference to edge-water intrusion. He presents a theoretical study on time-way relation of movement of liquid-liquid interface, and on the effect of rate of withdrawal, etc. Problem was previously approached by M. Muskat [AMR **5,** Rev. 1904]. As no conclusive treatment seems to exist at present, covering the whole subject, author also refers to model tests now being carried out in the B.P.M. Laboratory in Amsterdam, and holds out the prospect of a report on results obtained thereby.

P. W. Werner, Sweden

3259. Cornell, D., and Katz, D. L., Pressure gradients in natural gas reservoirs, *J. Petr. Technol.* **5,** 3, 61–69, Mar. 1953.

Authors show that the Schmidt method employed in heat conduction is readily applicable when calculating pressure gradients in certain gas reservoirs. M. J. Goglia, USA

Micromeritics

3260. Rosenberg, B., The motion of small slender particles in a viscous incompressible fluid, Proc. First U. S. nat. Congr. appl. Mech., June 1951; J. W. Edwards, Ann Arbor, Mich., 807–811, 1952.

The motion of slender particles of colloidal size under the action of viscous forces in two-dimensional flow is discussed. It is found that the analysis used to obtain the orientation of these particles in Couette flow is not sufficiently general to be applied to all types of two-dimensional flow. Instead, for any but parallel flow, there are two equilibrium positions. For diverging flow, the streamline is not the position of stable equilibrium.

From author's summary by J. S. McNown, USA

3261. Brun, R. J., and Mergler, H. W., Impingement of water droplets on a cylinder in an incompressible flow field and evaluation of rotating multicylinder method for measurement of droplet-size distribution, volume-median droplet size, and liquid-water content in clouds, *NACA TN* 2904, 71 pp., Mar. 1953.

Evaluation of the rotating multicylinder method for the measurement of droplet-size distribution, volume-median droplet size, and liquid-water content in clouds show that small uncertainties in the basic data eliminate the distinction between different cloud droplet-size distributions and are a source of large errors in the determination of the droplet size. Calculations of the trajectories of cloud droplets in an incompressible air flow field around a cylinder were performed on a mechanical analog constructed for the study of the trajectories of droplets around aerodynamic bodies.

Matching curves for obtaining droplet-size distribution, volume-median droplet size, and liquid-water content from flight data were computed from the results of the droplet-trajectory calculations. An evaluation is presented of the rotating multicylinder method for the measurement of droplet-size distribution, volume-median droplet size, and liquid-water content in clouds. Because of the insensitivity of the multicylinder method to changes in conditions in clouds, and the inaccuracies in obtaining flight data, errors as large as 70% in the determination of the volume-median droplet size are possible if the flight speed is 200 mph and the actual volume-median droplet diameter in the cloud is 30 microns. From authors' summary

3262. Manley, R. St. J., and Mason, S. G., Particle motions in sheared suspensions. II. Collisions of uniform spheres, *J. Colloid Sci.* **7,** 4, 354–369, Aug. 1952.

The interaction of model glass spheres in suspensions subjected to velocity gradients has been studied in detail. Absolute inelastic collision frequencies have been measured and found to be in excellent accord with a theory based upon simple geometrical considerations.

A doublet so formed rotates as a rigid dumbbell at a known constant angular velocity and separates at a point which is a mirror image of the initial point of contact. This memory effect persists through interruptions of motion of the suspension, a phenomenon which casts some doubt on the otherwise definite evidence of true contact of the spheres.

The details of doublet rotation which have been established experimentally make it possible to calculate the mean and the maximum life of a doublet, and the frequency distribution of

doublet life. Good agreement between various measured and calculated values of doublet life has been found.

From authors' summary by L. J. Tison, USA

Geophysics, Meteorology, Oceanography

(See also Revs. 3005, 3125)

3263. Isimaru, Y., The mechanism of the earth's atmosphere. On the general atmospheric circulation, *Geophys. Mag., Tokyo* **23,** 3, 229–301, Mar. 1952.

Author presents scholarly and comprehensive treatment of the various equations governing atmospheric phenomena observed in the troposphere. This work is a generalization and extension of Oberbeck's theory on general circulation of the atmosphere.

Derivations are given for particle motions and temperature, pressure, and density distributions in the lower atmosphere. Through the use of certain assumptions, semiclosed solutions to the equations are obtained for a wide variety of conditions. As examples, the temperature, density, and pressure distributions are obtained for the static and dynamic compressible atmosphere; numerical expressions for most general atmospheric circulation are given; zonal wind distributions are calculated; etc.

Author develops atmospheric motions on the basis of principal and secondary circulations. Principal circulation arises from particles driven by thermal and gravitational forces, whereas secondary circulation is the result of a variable density distribution. Combining these circulations, author obtains good qualitative agreement with observed tropospheric circulation– including jet stream position.

Paper is not elementary. If criticism is given, it falls on an occasional lack of clarity in exposition and the omission of a much-needed glossary for ready reference.

W. W. Berning, USA

3264. van Mieghem, J., Energy conversions in the atmosphere on the scale of the general circulation, *Tellus* **4,** 4, 334–351, Nov. 1952.

Balance equations of the potential, kinetic, and internal energies are derived and expressed in terms representing the fluxes, rates of production, and conversions between different energy forms. These balance equations are simplified by introducing mean values, weighted with respect to density and averaged along the latitude circles, which can be applied to the study of atmospheric motions of the scale directly related to the general circulation. The study shows that the mean zonal motion derives its kinetic energy mainly from kinetic energy of large-scale disturbances, through the meridional flux of relative angular momentum and the meridional shear of the mean zonal wind, while the large-scale disturbances derive their kinetic energy mainly from the conversion of potential energy into kinetic energy of these large eddies. Large-scale heat diffusion is also discussed. As these energy transformations are the most fundamental processes of these large-scale motions, they should be taken as the bases or guides for more detailed studies of the dynamical aspects of these motions.

H. L. Kuo, USA

3265. van Mieghem, J., Some remarks on the angular momentum balance in the atmosphere, *Tellus* **4,** 2, 135–138, May 1952.

The author writes down in spherical coordinates the equations satisfied by the angular momentum of a viscous fluid. He averages these equations along a circle of latitude and discusses the interpretation and magnitude of the terms occurring.

C. Truesdell, USA

3266. Høiland, E., On horizontal motion in a rotating fluid, *Geofys. Publ. Norske Vid.-Akad. Cslo* **17,** no. 10, 26 pp., 1950.

An integral treatment of the velocity circulation for horizontal motions of the atmosphere leads the author to conclude that, neglecting baroclinicity, the vertical component of the absolute vorticity of an element of air is constant "following the motion." This conclusion also depends on the implicit assumption that the air is incompressible horizontally, so that the equation of continuity is satisfied by the introduction of a stream function. After some general discussion of the phenomena in cyclones and anticyclones arising from the constancy of the absolute vorticity component, the author passes to motions in which the vertical component of absolute vorticity is proportional to the stream function. The differential equation for this function is solved in a number of cases, allowance being made for the sphericity of the earth. The solutions are described in detail with the help of diagrams. A brief treatment of the impulse theorem is followed by solutions of a linearized equation for the wave motions produced by perturbations of a constant zonal flow. Waves in a rectangular basin are also referred to.

Courtesy of Mathematical Reviews G. C. McVittie

3267. Hovmöller, E., A comparison between observed and computed winds with respect to their applicability for vorticity computations, *Tellus*, **4,** 2, 126–134, May 1952.

First, the method of computation of the geostrophic wind field and the vorticity field based on the topography of a constant-pressure surface is discussed. Then the method of computation of the vorticity field based on unobserved winds is discussed. (The reviewer would like to cite an independent but antecedent work by A. H. Gordon [*Quart. J. roy. meteor. Soc.* **78,** 335, 85–88, Jan. 1952] who developed the same procedure.) Lastly, the procedure to compute the vorticity field based on all available data is described as a recommendation.

H. Arakawa, Japan

3268. Chapman, J. H., A study of winds in the ionosphere by radio methods, *Canad. J. Phys.* **31,** 1, 120–131, Jan. 1953.

About 500 observations in *E* and *F* regions were made in eastern Canada using time variations of the fading of radio signals in a network of three receivers arranged in a triangle. The modal wind speed was 80 mps with maxima of 300 mps.

A semi-diurnal course of wind speed and direction was discovered; this oscillation is thought to be connected with the solar and lunar tides. At least, in the *F* layer a positive correlation exists between wind speed and the *K*-index of magnetic activity.

H. Riehl, USA

3269. Homma, S., Love waves in a surface layer of varying thickness, *Geophys. Mag., Tokyo* **24,** 1, 9–14, Aug. 1952.

Author considers asymptotic expressions for particular solutions of wave equation in a surface layer of linearly varying thickness, and concludes that a Love wave can be transmitted in an approximate sense. Distance of transmission without much change in wave form depends on wave length. Problem is of interest in connection with seismic waves.

L. Malvern, USA

3270. Keilis-Borok, V. I., On the frequency equation of a multilayer elastic medium (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)* **87,** 1, 25–28, Nov. 1952.

Investigating the frequency equation for the half space, consisting of several finite thickness layers and a half space, the theorem is proved that, for sufficiently small total thickness of the finite layers, there exists one simple real positive root. Hence,

free as well as forced vibration may occur, where the latter will predominate at large distances from the source of disturbance.
J. R. M. Radok, Australia

3271. **Rikitake, T., Electrical conductivity and temperature in the earth,** *Bull. Earthq. Res. Inst., Tokyo* **30,** 1, 13–23, Mar. 1952.

Author revises his previous treatment [title source, **27,** p. 17, 1949] of the problem. In the present analyses, the assumptions of compressibility independent of pressure and that the temperature in the earth increases proportionally to depth are eliminated. In addition, the pressure effect on the activation energy of the motion of ions is examined. The results disagree with works of Bullard and Jeffreys but are in fair agreement with the work of Gutenberg. R. J. Mindak, USA

3272. **Tolstoy, I., Edwards, R. S., and Ewing, M., Seismic refraction measurements in the Atlantic Ocean,** *Bull. seism. Soc. Amer.* **43,** 1, 35–48, Jan. 1953.

Four fully or partly reversed seismic refraction measurements were made during the winter of 1950 in the North Atlantic Basin by the whaleboat method. From authors' summary

3273. **Longuet-Higgins, M. S., On the statistical distribution of the heights of sea waves,** *J. mar. Res.* **11,** 3, 245–266, 1952.

The statistical distribution of wave heights is derived theoretically on the assumptions (a) that the wave spectrum contains a single narrow band of frequencies, and (b) that the wave energy is being received from a large number of different sources whose phases are random. Theoretical relations are found between the root-mean-square wave height, the mean height of the highest one third (or highest one tenth) waves, and the most probable height of the largest wave in a given interval of time. There is close agreement with observation.
From author's summary by W. H. Munk, USA

3274. **Keulegan, G. H., Hydrodynamic effects of gales on Lake Erie,** *J. Res. nat. Bur. Stands.* **50,** 2, 99–110, Feb. 1953.

On the basis of simultaneous measurements of wind speed and the resulting displacement of water level, the coefficient of wind stress $x = \tau/\rho_a V^2$ (τ stress, ρ_a air density, V wind speed) is derived for 22 gales with wind speeds up to 51 mph. The results compare favorably with Neumann's observations in the Gulf of Bothnia. Unfortunately, winds were observed at different levels, and the reduction to a standard elevation (why 150 ft?) introduces some uncertainty. Reviewer would have welcomed a discussion of the possible role played by the lake's thermal structure, as well as a comparison with the somewhat different results obtained in the author's excellent laboratory determination of the coefficient of wind stress [AMR **4,** Rev. 3488]. W. H. Munk, USA

Lubrication; Bearings; Wear

3275. **Bukowiecki, A., On the specific requirements of the corrosion testing of lubricants** (in German), *Schweiz. Arch.* **19,** 1, 22–31, Jan. 1953.

In experiments with several types of lubricating oils, it has been found that conventional analytical methods such as the determination of the neutralization number, or the water, chlorine, and sulphur contents do not suffice to accurately describe their corroding properties. In addition, actual corrosion tests are therefore necessary and can generally be carried out without undue difficulties. They chiefly comprise: (1) A stirring test with heated automotive crankcase oils; (2) a test in glass disks with water-in-oil dispersions; (3) two methods with metal specimen covered with only a thin surface layer of oil (drop test and exposure to acid vapors); (4) an after-rusting test with automotive gear oils.
From author's summary by F. T. Barwell, Scotland

3276. **Niemann, G., and Banaschek, K., Friction coefficients of lubricated surfaces** (in German), *ZVDI* **95,** 6, 167–173, Feb. 1953.

Experiments are described wherein cylindrical pads were forced against the sides of a disk in the presence of lubricant. One pad was arranged at each side of the disk so that the reaction was balanced. Measurements were made of frictional force, the temperature of the oil at outlet and the electrical resistance of the oil film under a potential difference of two volts. Pads of radius varying from $^1/_2$ to 10 cm were made from three different materials giving a range of hardness and surface finish. Loads ranged from 0.5 to 75 kg/cm width (2.8 to 420 lb/in. width). Two lubricants were used, one being of the hypoid type. Results are set out in a form of curves of coefficient of friction against rubbing speed, the maximum rubbing speed being approximately 1300 cm/sec (42.6 fps). At low loads frictional drag increases rapidly with rubbing speed, but at higher loads remains constant and even diminishes slightly. This is probably due to falling off in viscosity accompanying temperature rise. Frictional drag increases with load although coefficient of friction generally falls off. No attempt is made to present the results in a form of nondimensional parameters.

Other curves connect the minimum coefficient of friction with the square root of the reciprocal of the radius of curvature of pad. Comparison of turned, ground, and polished material shows that the turned material gives a much higher minimum coefficient of friction. A plot of minimum coefficient of friction multiplied by the square root of the radius of curvature against the square root of the surface roughness shows a linear increase. This method of plotting reveals only a slight difference between various oils. Curves connecting initial or static coefficient of friction with load for various radii of curvature purport to show that sharper radius gave the lowest friction. The difference between the results reported for the different radii is, however, slight compared with scatter of the results, which is normal for this type of test.

Reviewer believes the results are worthy of further study on a nondimensional basis, but comparison with theory is rendered difficult because of ambiguity regarding the boundary conditions governing film formation. These would probably vary with the amount of lubricant supplied. F. T. Barwell, Scotland

3277. **Osterle, F., Charnes, A., and Saibel, E., On the solution of the Reynolds-equation for slider-bearing lubrication—VI. The parallel surface slider bearing without side leakage,** ASME Ann. Meet., New York, Dec. 1952. Paper 52—A-33, 10 pp., 1 fig.

Paper deals with problem of Fogg's "thermal wedge" effect in a parallel film bounded by rubbing surfaces that are assumed not to absorb any of the viscous-frictional heat. Authors combine Reynolds equation, which is generalized for nonuniform oil density, with a linear density-temperature relationsip, an exponential viscosity-temperature-pressure relationship, and the energy equation, in which internal energy of the oil is considered to be independent of pressure and to increase linearly with temperature.

No exact solution is given for the distribution of temperature and pressure along the film, but it was possible to set upper and lower bounds to them. In the numerical example treated, in which film pressures are only moderate so that pressure variation of viscosity is negligible, the bounds are so close together that

their average yields a good approximation to the unknown exact solution. H. Blok, Holland

3278. Blackburn, J. F., Contributions to hydraulic control. 5—Lateral forces on hydraulic pistons, ASME Ann. Meet., New York, Dec. 1952. Paper 52—A-44, 12 pp., 6 figs.

Classical Reynolds theory of lubrication is applied by author to study transverse forces on piston lands. Mathematical solution is given for decentered tapered piston inside cylinder bore, using Reynolds formulas for block sliding over plane surface. These formulas are exhibited and plotted in dimensionless form.

Plots of experiments, both for tapered and stepped piston lands, are reported, showing good agreement with theory, especially for tapered piston and small values of decentration. Short qualitative analysis follows on other schemes of piston shape and their physical correspondences. Paper closes with general remarks on lubrication problems.

G. Evangelisti, Italy

3279. Frenkel, M. S., Oscillatory occurrences in ball bearings, *J. roy. aero. Soc.* **56,** 504, 885–908, Dec. 1952.

When an angular contact bearing is subjected to analytical treatment it is found that, in general, an oscillatory component of motion exists in addition to the desired relative motion between balls and races. This oscillatory motion may cause premature fatigue and increased friction loss and, hence, is to be avoided. The oscillatory motion is shown to be a function of speed, load, and bearing dimensions. For a given speed and load there is a set of dimensions for which the oscillatory motion becomes zero. Relations are given from which a set of dimensions may be determined to yield a bearing free of oscillatory motion.

M. C. Shaw, USA

3280. Sines, G. H., The dynamics and lubrication of a miniature turbine rotor on journal bearings, *Univ. Calif., Dept. Engng. Rep.* 52.13, 58 pp., June 1952.

Paper describes an experimental investigation of the motion of a $^3/_{16}$-in. diam journal in the clearance space of a porous sintered bearing. The variables covered include angular velocities to 90,000 rpm, balanced and unbalanced shafts, bearing clearances, lubricants with and without boundary lubrication additives, quantity of lubricant present, and degree of surface porosity. It is found that the journal may operate in a fixed position, may undergo stable orbital motion in a circular path, or erratic orbital motion, depending on the choice of the above variables. Mechanisms for each case are advanced, as is an interesting comparison of the porous bearing with a solid partial bearing of small angular extent.

An appendix contains a useful analysis of the optical system used to record the motion of the journal center in the experiments.

C. D. Strang, Jr., USA

3281. Poritsky, H., Contribution to the theory of oil whip, ASME Ann. Meet., New York, Dec. 1952. Paper 52—A-64, 20 pp.

It is known that under certain conditions a rotor shaft mounted in sleeve bearings whips when the rotor is running at any speed above double critical speed. Author shows that an explanation of oil-whip phenomena, at least for small eccentricities, can be obtained from the equations of hydrodynamic forces and the dynamical equations of motion, provided that the hydrodynamic-force expressions be modified to delete from oil forces the hitherto included contributions from regions of negative pressure. While exact determination is rather difficult to carry out, an approximate procedure gives satisfactory account of oil-whip phenomena for small eccentricities; there is added a force component in the radial direction, linear in radial displacement and with a proper "stiffness constant."

For a vertical rotor in which the asymmetry caused by gravity load does not occur, small-displacement theory yields, on the strength of well-known criteria for stability or instability, that oil forces lead to an unstable system and that center displacement, if not exactly zero initially, will grow with time without a stop, no matter what rotational speed of rotor is. If the basic equations are modified by addition of a radial component of force on the journal, linear in displacement of journal center from bearing center, then stability is restored for speeds below double the critical speed, while at speeds above double the critical speed, instability persists. It is further shown that under the same condition, the effect of gravity load of rotor may be taken into account and does not affect stability or instability conditions.

Finally, displacements due to applied periodic forces on rotating journal—the "elastic behavior" of an oil film—are studied. Then journal center describes an ellipse whose principal axes are inclined relative to the (assumed horizontal) direction of applied periodic force. The system will be stable or unstable according to whether the former system is stable or not.

G. Vogelpohl, Germany

Marine Engineering Problems

3282. Weinblum, G. P., Kendrick, Janet J., and Todd, M. A., Investigation of wave effects produced by a thin body—TMB model 4125, *David W. Taylor Mod. Basin Rep.* 840, 19 pp., Nov. 1952.

A comparison between "thin ship" theory and experiment for the wave resistance $C_w(F)$ and wave profiles of a hull with parallel middle body, and parabolic ends with vertical sides. Agreement is "satisfactory"; as usual, calculated "humps and hollows" in $C_w(F)$ are more extreme than those measured, and predicted wave crests and troughs are somewhat aft of those observed. G. Birkhoff, USA

3283. Balhan, J., Some considerations on the cavitation and erosion of ship propellers (in Dutch), *Ingenieur* **65,** 6, 7; W.29–W.40, W.41–W.43; Feb. 1953.

From tests on propeller models in a cavitation tunnel, the following conclusions are drawn relative to cavitation erosion: Burbling cavitation, which is associated with thick profiles having well-rounded leading edges, does not lead to erosion. Sheet cavitation, associated with thin profiles, causes erosion when clouds of minute bubbles are formed in the downstream region of the sheet. Intermittent cavitation in a nonuniform velocity field promotes the formation of clouds and of ensuing erosion.

H. W. Lerbs, USA